Representations of Groups

Math 6510, Fall 2009, MW 12:30-1:50, Business 231

Professor: Dr. Conley, GAB 419, 565-3326, conley@unt.edu. Assignments and announcements will be posted at www.math.unt.edu/∼conley.

Office Hours: MW 11:00-12:30, F 11:00-12:00

Grades: based on attendance and problem sets.

Prerequisites: The most important prerequisites are linear algebra (at the level of 4450 and 5530) and abstract group theory (at the level of 5520). Vector calculus (3740) and manifold theory (5450) may occasionally be helpful, but they will not be necessary.

References: There will be no formal text. At times I will draw from "Representation Theory, A First Course," by Fulton and Harris, and "Group Theory and Physics," by Sternberg. There are many introductions to the subject: some examples on the algebraic side are those by Humphreys, Goodman and Wallach, and Jacobson, and on the analytic side, those by Varadarajan, Bröcker and tom Dieck, and Sepanski.

Plan: Attendance permitting, the course will run for two semesters. The first part (ten weeks or so) will concern representations of finite groups. We will begin with elementary definitions, using S_3 as an example. We will then proceed to character theory, orthogonality relations, the fact that there are as many irreducible representations as conjugacy classes, and the decomposition of the regular representation. En route we will consider various examples, such as the cyclic and dihedral groups C_n and D_n, the alternating group A_4, and the symmetric group S_5 (we probably will not have time for S_n, but if you have an interest in this, please let me know). We will also treat duals and tensor products and touch on finite Fourier transforms and vibrational modes of molecules.

The second part of the course will be an introduction to compact Lie groups and their representations. We will work with groups of matrices, beginning with the rank 1 case (SU_2 and its relatives SO_3, O_3, and U_2) and then moving to the simplest rank n case, SU_{n+1} (with emphasis on the rank 2 case SU_3). In this setting we will see again the ingredients of the first part of the course, along with some new ones: the exponential map, invariant integration, Lie algebras, and the Peter-Weyl theorem.

We will focus on Cartan's "infinitesimal method": the classification of the irreducible representations of the relevant Lie algebras via root systems, Weyl groups, and the method of the highest weight. If time permits we will cover the Clebsch-Gordan rules, Kostant's formula, Steinberg's formula, the Weyl character and dimension formulas, and, at least for SU_2, Weyl's "transcendental method". If time still permits, we will glance at the multiply laced rank 2 cases SO_5 and G_2.