

I was discussing the general partial fraction form for the case of the denominator $q(x)$ factoring fully into real linear factors. I wrote out the factorization of $q(x)$ as

$$q(x) = c(x - c_1)^{m_1}(x - c_2)^{m_2} \dots (x - c_k)^{m_k},$$

where we're assuming c and each c_i are real numbers and each m_i is a positive integer. (This factorization is really into *powers of* real linear factors, but if we can factor to this form then we can also literally factor to a product of linear factors, by writing each power $(x - c_i)^{m_i}$ out as $(x - c_i)(x - c_i) \dots (x - c_i)$.)

I was allowing $q(x)$ to have repeated roots, so it's possible to have some (or all) of the exponents $m_i > 1$. So the roots for $q(x)$ are c_1, c_2, \dots, c_k . In class I said something like that these roots c_i were listed "without repetition" (i.e. that for $i \neq j$ we have $c_i \neq c_j$). This might have sounded like I was saying $q(x)$ had no repeated roots, but it doesn't say this. It just means that each root is listed once, no matter how many times it's actually repeated as a root of $q(x)$, i.e. no matter what the powers m_i are. So, e.g., if $q(x) = x^3(x - 2)^5(x + 7)$, then q has 3 distinct roots, $c_1 = 0$, $c_2 = 2$, and $c_3 = -7$. So the list has just $k = 3$ elements, with each root listed just once. (Even though $c_1 = 0$ and $c_2 = 2$ are repeated roots of $q(x)$.)

Note that if in this setup (with q 's roots being c_1, c_2, \dots, c_k , each listed only once), we have that m_i is the multiplicity of c_i , and c_i is a repeated root of $q(x)$ iff $m_i > 1$.