Intro to topology homework 13. Due Friday May 4.
Please give to Steve Jackson.

From Munkres:
§24: 8(note), 10

In §24: 8, part (b) is not required, just for extra credit. In it, $\bar{A} = \mathrm{Cl}(A)$. A hint for it: there are spaces X with a subset $A \subseteq X$ which is sequentially closed, but not closed, and such that $X \backslash A$ is closed; you may assume this.

1.(a) Let X be a top space and $\langle x_n \rangle_{n \in \mathbb{N}}$ be a convergent sequence of points in X, converging to x. Let $C = \{x_n\}_{n \in \mathbb{N}} \cup \{x\}$. Prove that C is compact.

(b) Prove that for any set X, if $\tau = \tau_{\text{cof}}$ is the cofinite topology on X, then every subset of X is compact with respect to τ.

2. Suppose A, B are compact subsets of a top space (X, τ).

(a) Prove that $A \cup B$ is compact.

(b) Prove that if X is T_2 then $A \cap B$ is compact. (Remark: There are counterexamples when X isn't required to be T_2.)

3. Prove that there is no continuous surjective function $f : [0,1] \times [0,1] \to \mathbb{R}$ (with the standard topology on $[0,1] \times [0,1] \subseteq \mathbb{R}^2$).

4.(4500; you can do the 5600 version for extra credit) Let $A \subseteq \mathbb{B}((0,0,0),1) \subseteq \mathbb{R}^3$ with $A \neq \emptyset$ and A closed. Prove that there are finitely many points $a_1, \ldots, a_n \in A$ such that for every $a \in A$, $d(a, a_i) < 0.01$ for some $i \in \{1, \ldots, n\}$.

4.(5600) Let $A \subseteq \mathbb{B}((0,0,0),1) \subseteq \mathbb{R}^3$ with $A \neq \emptyset$. Prove that there are finitely many points $a_1, \ldots, a_n \in A$ such that for every $a \in A$, there is $i \in \{1, \ldots, n\}$ such that $d(a, a_i) < 0.001$. (Hint: first assume A is closed, then generalize that to work when A is not closed.)