

Intro to Topology Homework 1B - due Thursday Jan 26.

Note: Problems with a * are those suitable for the homework discussion requirement.

1. Show that \mathbb{Q} is not a closed set.

2*(a) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function. Show that $\{x \in \mathbb{R} \mid f(x) > 0\}$ is an open set.

(b) Let $f, g, h : \mathbb{R} \rightarrow \mathbb{R}$ be continuous functions. Show that $\{x \in \mathbb{R} \mid f(x) > 0 \ \& \ g(x) > 0 \ \& \ h(x) < 1\}$ is an open set.

3(not required, bonus credit). In the proof given on Thursday of the fact that every open set is a union of open intervals, I used the Axiom of Choice (to choose an $\varepsilon_x > 0$ for a given $x \in A$). Show that the appeal to the Axiom of Choice was actually unnecessary here. (To do this, come up with a specific rule by which a suitable ε_x can be chosen (depending just on x and the open set A we started with). The Axiom of Choice is only necessary in situations in which there is no specific rule by which one can make choices.)