Exercise Set R.4

Graph.

1. $x = 3$

2. $x = 5$

3. $y = -2$

4. $y = -4$

5. $x = -4.5$

6. $x = -1.5$

7. $y = 3.75$

8. $y = 2.25$

Graph. List the slope and y-intercept.

9. $y = -2x$

10. $y = -3x$

11. $f(x) = 0.5x$

12. $f(x) = -0.5x$

13. $y = 3x - 4$

14. $y = 2x - 5$

15. $g(x) = -x + 3$

16. $g(x) = x - 2.5$

17. $y = 7$

18. $y = -5$

Find the slope and y-intercept.

19. $y - 3x = 6$

20. $y - 4x = 1$

21. $2x + y - 3 = 0$

22. $2x - y + 3 = 0$

23. $2x + 2y + 8 = 0$

24. $3x - 3y + 6 = 0$

25. $x = 3y + 7$

26. $x = -4y + 3$

Find an equation of the line:

27. with $m = -5$, containing $(-2, -3)$.

28. with $m = 7$, containing $(1, 7)$.

29. with $m = -2$, containing $(2, 3)$.

30. with $m = -3$, containing $(5, -2)$.

31. with slope 2, containing $(3, 0)$.

32. with slope -5, containing $(5, 0)$.

33. with y-intercept $(0, -6)$ and slope $\frac{1}{2}$.

34. with y-intercept $(0, 7)$ and slope $\frac{4}{3}$.

35. with slope 0, containing $(2, 3)$.

36. with slope 0, containing $(4, 8)$.

Find the slope of the line containing the given pair of points. If a slope is undefined, state that fact.

37. $(5, -3)$ and $(-2, 1)$

38. $(-2, 1)$ and $(6, 3)$

39. $(2, -3)$ and $(-1, -4)$

40. $(-3, -5)$ and $(1, -6)$

41. $(3, -7)$ and $(3, -9)$

42. $(-4, 2)$ and $(-4, 10)$

43. $\left(\frac{4}{5}, -3\right)$ and $\left(\frac{1}{2}, \frac{2}{5}\right)$

44. $\left(-\frac{3}{16}, -\frac{1}{2}\right)$ and $\left(\frac{5}{8}, -\frac{3}{4}\right)$

45. $(2, 3)$ and $(-1, 3)$

46. $\left(-6, \frac{1}{2}\right)$ and $\left(-7, \frac{1}{2}\right)$

47. $(x, 3x)$ and $(x + h, 3(x + h))$

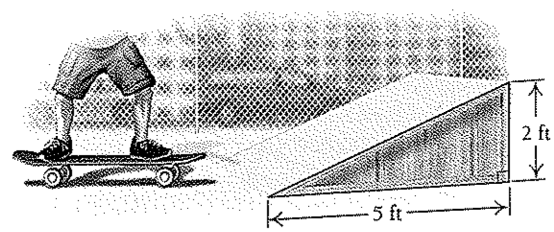
48. $(x, 4x)$ and $(x + h, 4(x + h))$

49. $(x, 2x + 3)$ and $(x + h, 2(x + h) + 3)$

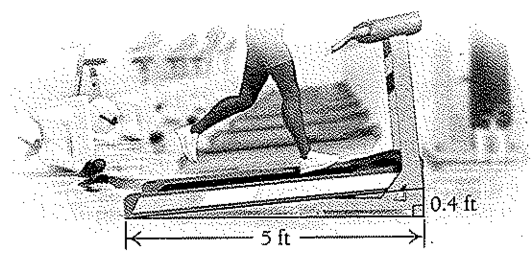
50. $(x, 3x - 1)$ and $(x + h, 3(x + h) - 1)$

51–60. Find an equation of the line containing the pair of points in each of Exercises 37–46.

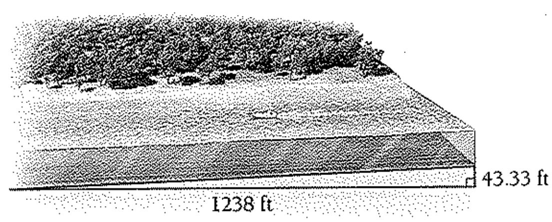
61. Find the slope of the skateboard ramp.



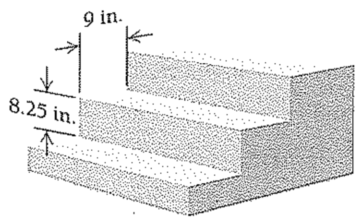
62. Find the slope (or grade) of the treadmill.



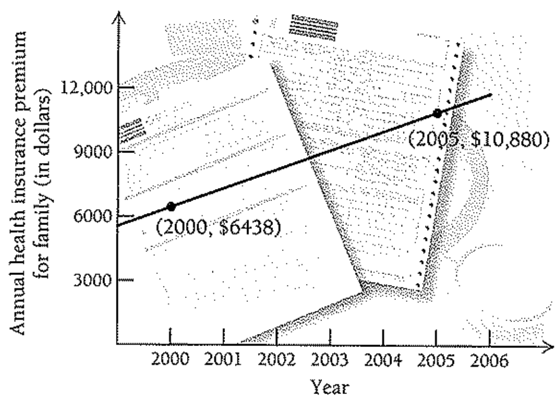
63. Find the slope (or head) of the river. Express the answer as a percentage.



64. Stair requirements. A North Carolina state law requires that stairs have minimum treads of 9 in. and maximum risers of 8.25 in. (*Source:* North Carolina Office of the State Fire Marshal.) See the illustration below. According to this law, what is the maximum grade of stairs in North Carolina?

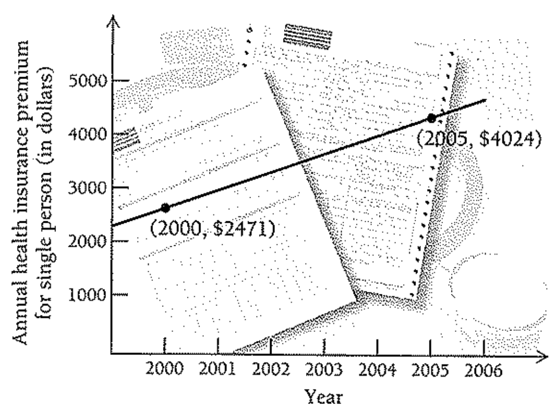


65. Health insurance premiums. Find the average rate of change in the annual premium for a family's health insurance.



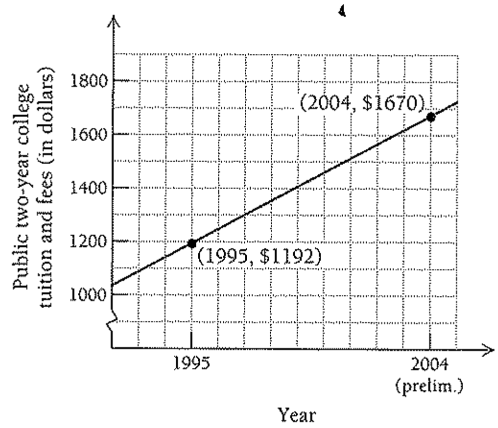
(*Source:* The Kaiser Family Foundation; Health Research and Education Trust.)

66. Health insurance premiums. Find the average rate of change in the annual premium for a single person.



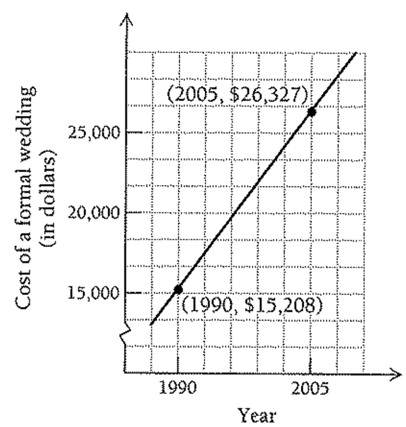
(*Source:* The Kaiser Family Foundation; Health Research and Education Trust.)

67. Find the average rate of change of the tuition and fees at public two-year colleges.



(*Source:* U.S. National Center for Education Statistics, *Digest of Education Statistics*, annual.)

68. Find the average rate of change of the cost of a formal wedding.



(*Source:* The Fairchild Bridal Group.)

69. Energy conservation. The R-factor of home insulation is directly proportional to its thickness T.

a) Find an equation of variation if $R = 12.51$ when $T = 3$ in.

b) What is the R-factor for insulation that is 6 in. thick?

70. Nerve impulse speed. Impulses in nerve fibers travel at a speed of 293 ft/sec. The distance D, in feet, traveled in t sec is given by $D = 293t$. How long would it take an impulse to travel from the brain to the toes of a person who is 6 ft tall?

71. Muscle weight. The weight M of the muscles in a human is directly proportional to the person's body weight W.

EXAMPLE 15 Economics: Equilibrium Point. Find the equilibrium point for the demand and supply functions for the Ultra-Fine coffee maker. Here q represents the number of coffee makers produced, in hundreds, and x is the price, in dollars.

Demand: $q = 50 - \frac{1}{4}x$

Supply: $q = x - 25$

Solution To find the equilibrium point, the quantity demanded must match the quantity produced:

$$50 - \frac{1}{4}x = x - 25$$

$$50 + 25 = x + \frac{1}{4}x \quad \text{Adding } 25 + \frac{1}{4}x \text{ to each side}$$

$$75 = \frac{5}{4}x$$

$$75 \cdot \frac{4}{5} = x \quad \text{Multiplying both sides by } \frac{4}{5}$$

$$60 = x.$$

Thus, $x_E = 60$. To find q_E, we substitute x_E into either function. We select the supply function:

$$q_E = x_E - 25 = 60 - 25 = 35.$$

Thus, the equilibrium quantity is 3500 units, and the equilibrium point is (\$60, 3500). ◆

TECHNOLOGY CONNECTION

EXERCISE

1. Use the INTERSECT feature to find the equilibrium point for the following demand and supply functions.

   Demand: $q = 1123.6 - 61.4x$
   Supply: $q = 201.8 + 4.6x$

Exercise Set R.5

Graph each pair of equations on one set of axes.

1. $y = \frac{1}{2}x^2$ and $y = -\frac{1}{2}x^2$

2. $y = \frac{1}{4}x^2$ and $y = -\frac{1}{4}x^2$

3. $y = x^2$ and $y = x^2 - 1$

4. $y = x^2$ and $y = x^2 - 3$

5. $y = -2x^2$ and $y = -2x^2 + 1$

6. $y = -3x^2$ and $y = -3x^2 + 2$

7. $y = |x|$ and $y = |x - 3|$

8. $y = |x|$ and $y = |x - 1|$

9. $y = x^3$ and $y = x^3 + 2$

10. $y = x^3$ and $y = x^3 + 1$

11. $y = \sqrt{x}$ and $y = \sqrt{x - 1}$

12. $y = \sqrt{x}$ and $y = \sqrt{x - 2}$

For each of the following, state whether the graph of the function is a parabola. If the graph is a parabola, find the parabola's vertex.

13. $f(x) = x^2 + 4x - 7$

14. $f(x) = x^3 - 2x + 3$

15. $g(x) = 2x^4 - 4x^2 - 3$

16. $g(x) = 3x^2 - 6x$

Graph.

17. $y = x^2 - 4x + 3$

18. $y = x^2 - 6x + 5$

19. $y = -x^2 + 2x - 1$

20. $y = -x^2 - x + 6$

21. $f(x) = 2x^2 - 6x + 1$

22. $f(x) = 3x^2 - 6x + 4$

23. $g(x) = -3x^2 - 4x + 5$

24. $g(x) = -2x^2 - 3x + 7$

25. $y = \frac{2}{x}$

26. $y = \frac{3}{x}$

27. $y = -\frac{2}{x}$

28. $y = \frac{-3}{x}$

29. $y = \frac{1}{x^2}$

30. $y = \frac{1}{x - 1}$

31. $y = \sqrt[3]{x}$

32. $y = \frac{1}{|x|}$

33. $f(x) = \frac{x^2 + 5x + 6}{x + 3}$

34. $g(x) = \frac{x^2 + 7x + 10}{x + 2}$

35. $f(x) = \frac{x^2 - 1}{x - 1}$

36. $g(x) = \frac{x^2 - 25}{x - 5}$

Solve.

37. $x^2 - 2x = 2$

38. $x^2 - 2x + 1 = 5$

39. $x^2 + 6x = 1$

40. $x^2 + 4x = 3$

41. $4x^2 = 4x + 1$

42. $-4x^2 = 4x - 1$

43. $3y^2 + 8y + 2 = 0$

44. $2p^2 - 5p = 1$

45. $x + 7 + \frac{9}{x} = 0$ (*Hint:* Multiply both sides by x.)

46. $1 - \frac{1}{w} = \frac{1}{w^2}$

Rewrite each of the following as an equivalent expression with rational exponents.

47. $\sqrt{x^3}$

48. $\sqrt{x^5}$

49. $\sqrt[5]{a^3}$

50. $\sqrt[4]{b^2}, \quad b \geq 0$

51. $\sqrt[7]{t}$

52. $\sqrt[8]{c}$

53. $\sqrt[4]{x^{12}}, \quad x \geq 0$

54. $\sqrt[3]{t^6}$

55. $\frac{1}{\sqrt{t^5}}$

56. $\frac{1}{\sqrt{m^4}}$

57. $\frac{1}{\sqrt{x^2 + 7}}$

58. $\sqrt{x^3 + 4}$

Rewrite each of the following as an equivalent expression using radical notation.

59. $x^{1/5}$

60. $t^{1/7}$

61. $y^{2/3}$

62. $t^{2/5}$

63. $t^{-2/5}$

64. $y^{-2/3}$

65. $b^{-1/3}$

66. $b^{-1/5}$

67. $e^{-17/6}$

68. $m^{-19/6}$

69. $(x^2 - 3)^{-1/2}$

70. $(y^2 + 7)^{-1/4}$

71. $\frac{1}{t^{2/3}}$

72. $\frac{1}{w^{-4/5}}$

Simplify.

73. $9^{3/2}$

74. $16^{5/2}$

75. $64^{2/3}$

76. $8^{2/3}$

77. $16^{3/4}$

78. $25^{5/2}$

Determine the domain of each function.

79. $f(x) = \frac{x^2 - 25}{x - 5}$

80. $f(x) = \frac{x^2 - 4}{x + 2}$

81. $f(x) = \frac{x^3}{x^2 - 5x + 6}$

82. $f(x) = \frac{x^4 + 7}{x^2 + 6x + 5}$

83. $f(x) = \sqrt{5x + 4}$

84. $f(x) = \sqrt{2x - 6}$

85. $f(x) = \sqrt[4]{7 - x}$

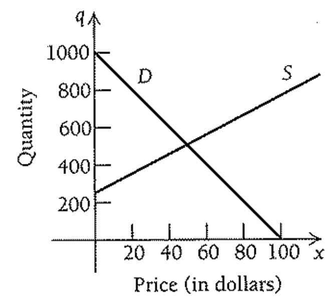
86. $f(x) = \sqrt[6]{5 - x}$

APPLICATIONS

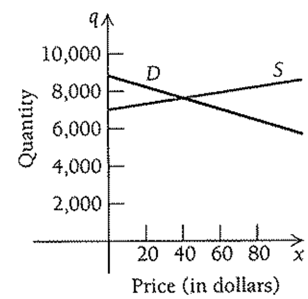
Business and Economics

Find the equilibrium point for each pair of demand and supply functions.

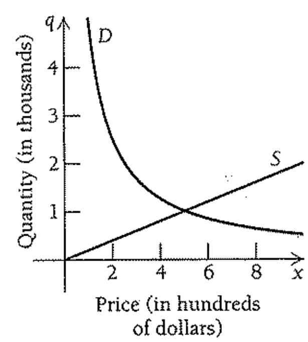
87. Demand: $q = 1000 - 10x$; Supply: $q = 250 + 5x$



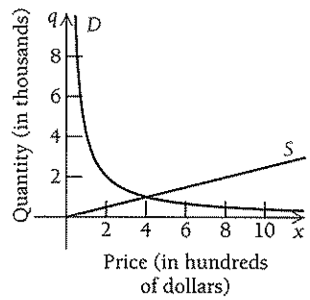
88. Demand: $q = 8800 - 30x$;
Supply: $q = 7000 + 15x$



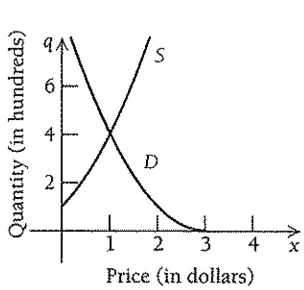
89. Demand: $q = \frac{5}{x}$; Supply: $q = \frac{x}{5}$



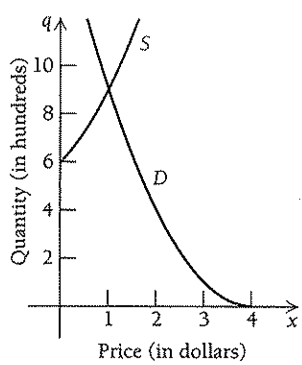
90. Demand: $q = \frac{4}{x}$; Supply: $q = \frac{x}{4}$



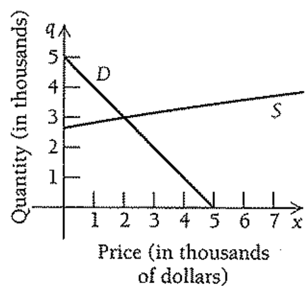
91. Demand: $q = (x - 3)^2$; Supply: $q = x^2 + 2x + 1$ (assume $x \leq 3$)



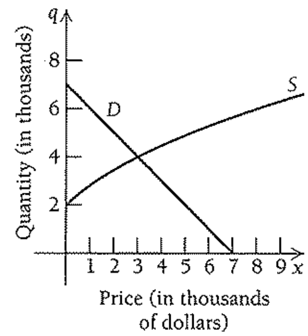
92. Demand: $q = (x - 4)^2$; Supply: $q = x^2 + 2x + 6$ (assume $x \leq 4$)



93. Demand: $q = 5 - x$; Supply: $q = \sqrt{x + 7}$



94. Demand: $q = 7 - x$; Supply: $q = 2\sqrt{x + 1}$



95. Stock prices and prime rate. It is theorized that the price per share of a stock is inversely proportional to the prime (interest) rate. In September 2005, the price per share S of Toyota stock was \$86.89 and the prime rate R was 6.75%. The prime rate rose to 7.50% in March 2006. (*Source:* finance.yahoo.com and Federal Reserve Board.) What would the price per share be if the assumption of inverse proportionality is correct?

96. Demand. The quantity sold x of a plasma television is inversely proportional to the price p. If 85,000 plasma TVs sold for \$2900 each, how many will be sold if the price is \$850 each?

97. Radar range. The function given by

$$R(x) = 11.74x^{0.25}$$

can be used to approximate the maximum range, $R(x)$, in miles, of an ARSR-3 surveillance radar with a peak power of x watts.

a) Determine the maximum radar range when the peak power is 40,000 watts, 50,000 watts, and 60,000 watts.

b) Graph the function.

98. Home range. Refer to Example 14. The home range, in hectares, of an omnivorous mammal (one that eats both plant and meat) of mass w grams is given by

$$H(w) = 0.059w^{0.92}.$$

(*Source:* Harestad, A. S., and Bunnel, F. L., "Home Range and Body Weight—A Reevaluation," *Ecology*, Vol. 10, No. 2 (April, 1979), pp. 389–402.) Complete the table of approximate function values and graph the function.

Solution As inputs x get larger and larger, outputs $f(x)$ get closer and closer to 3. We have

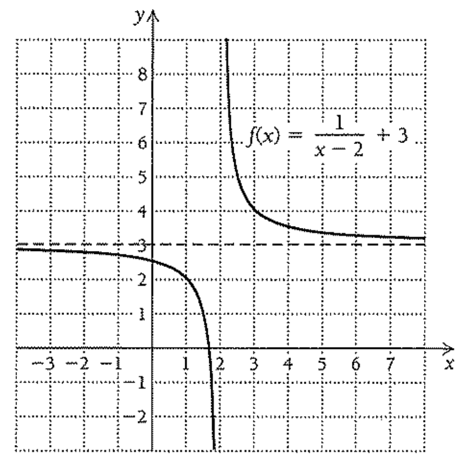
$$\lim_{x\to\infty} f(x) = 3.$$

Limit Numerically

$x \to \infty$	$f(x)$
5	$3.\overline{3}$
10	3.125
100	3.0102
1000	3.0010

$\longleftarrow \lim_{x\to\infty} f(x) = 3$

Limit Graphically



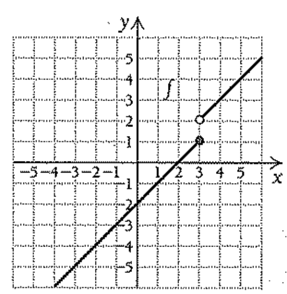
Exercise Set 1.1

Complete each of the following statements.

1. As x approaches 3, the value of $2x + 5$ approaches ________.
2. As x approaches -4, the value of $3x + 7$ approaches ________.
3. As x approaches ________, the value of $-3x$ approaches 6.
4. As x approaches ________, the value of $x - 2$ approaches 5.
5. The notation $\lim_{x\to 4} f(x)$ is read ________.
6. The notation $\lim_{x\to 1} g(x)$ is read ________.
7. The notation $\lim_{x\to 5^-} F(x)$ is read ________.
8. The notation $\lim_{x\to 4^+} G(x)$ is read ________.
9. The notation ________ is read "the limit, as x approaches 2 from the right."
10. The notation ________ is read "the limit, as x approaches 3 from the left."

For Exercises 11–18, consider the function f given by

$$f(x) = \begin{cases} x - 2, & \text{for } x \le 3, \\ x - 1, & \text{for } x > 3. \end{cases}$$



When necessary, state that the limit does not exist.

oser to 3.

11. Find $\lim_{x \to 3^+} f(x)$.

12. Find $\lim_{x \to 3^-} f(x)$.

13. Find $\lim_{x \to -1^-} f(x)$.

14. Find $\lim_{x \to -1^+} f(x)$.

15. Find $\lim_{x \to 3} f(x)$.

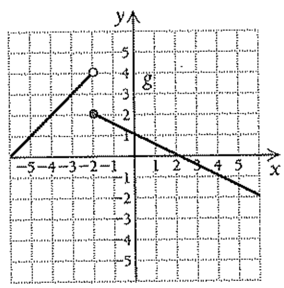
16. Find $\lim_{x \to -1} f(x)$.

17. Find $\lim_{x \to 4} f(x)$.

18. Find $\lim_{x \to 2} f(x)$.

For Exercises 19–26, consider the function g given by

$$g(x) = \begin{cases} x + 6, & \text{for } x < -2, \\ -\frac{1}{2}x + 1, & \text{for } x \geq -2. \end{cases}$$



If a limit does not exist, state that fact.

19. $\lim_{x \to -2^-} g(x)$

20. $\lim_{x \to -2^+} g(x)$

21. $\lim_{x \to 4^+} g(x)$

22. $\lim_{x \to 4^-} g(x)$

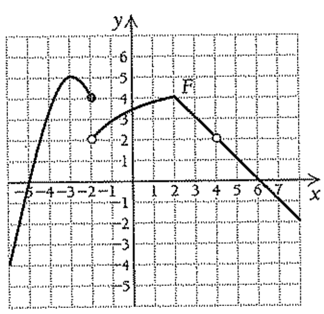
23. $\lim_{x \to 4} g(x)$

24. $\lim_{x \to -2} g(x)$

25. $\lim_{x \to 2} g(x)$

26. $\lim_{x \to -4} g(x)$

For Exercises 27–34, use the following graph of F to find each limit. When necessary, state that the limit does not exist.



27. $\lim_{x \to -3} F(x)$

28. $\lim_{x \to 2} F(x)$

29. $\lim_{x \to -2} F(x)$

30. $\lim_{x \to -5} F(x)$

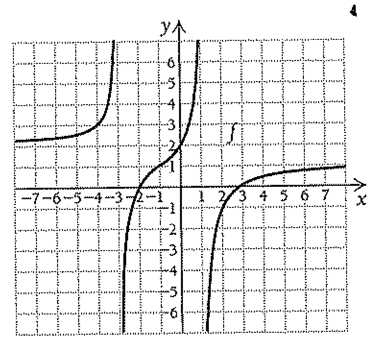
31. $\lim_{x \to 4} F(x)$

32. $\lim_{x \to 6} F(x)$

33. $\lim_{x \to -2^+} F(x)$

34. $\lim_{x \to -2^-} F(x)$

For Exercises 35–44, use the following graph of f to find each limit. When necessary, state that the limit does not exist.



35. $\lim_{x \to -1} f(x)$

36. $\lim_{x \to 2} f(x)$

37. $\lim_{x \to -3} f(x)$

38. $\lim_{x \to 0} f(x)$

39. $\lim_{x \to 3} f(x)$

40. $\lim_{x \to 1} f(x)$

41. $\lim_{x \to -4} f(x)$

42. $\lim_{x \to -2} f(x)$

43. $\lim_{x \to \infty} f(x)$

44. $\lim_{x \to -\infty} f(x)$

For Exercises 45–60, graph each function and then find the specified limits. When necessary, state that the limit does not exist.

45. $f(x) = |x|$; find $\lim_{x \to 0} f(x)$ and $\lim_{x \to -2} f(x)$.

46. $f(x) = x^2$; find $\lim_{x \to -1} f(x)$ and $\lim_{x \to 0} f(x)$.

47. $g(x) = x^2 - 5$; find $\lim_{x \to 0} g(x)$ and $\lim_{x \to -1} g(x)$.

48. $g(x) = |x| + 1$; find $\lim_{x \to -3} g(x)$ and $\lim_{x \to 0} g(x)$.

49. $F(x) = \dfrac{1}{x - 3}$; find $\lim_{x \to 3} F(x)$ and $\lim_{x \to 4} F(x)$.

50. $G(x) = \dfrac{1}{x + 2}$; find $\lim_{x \to -1} G(x)$ and $\lim_{x \to -2} G(x)$.

51. $f(x) = \dfrac{1}{x} - 2$; find $\lim_{x \to \infty} f(x)$ and $\lim_{x \to 0} f(x)$.

52. $f(x) = \dfrac{1}{x} + 3$; find $\lim_{x \to \infty} f(x)$ and $\lim_{x \to 0} f(x)$.

53. $g(x) = \dfrac{1}{x + 2} + 4$; find $\lim_{x \to \infty} g(x)$ and $\lim_{x \to -2} g(x)$.

54. $g(x) = \dfrac{1}{x - 3} + 2$; find $\lim_{x \to \infty} g(x)$ and $\lim_{x \to 3} g(x)$.

55. $F(x) = \begin{cases} 2x + 1, & \text{for } x < 1, \\ x, & \text{for } x \geq 1. \end{cases}$

Find $\lim_{x \to 1^-} F(x)$, $\lim_{x \to 1^+} F(x)$, and $\lim_{x \to 1} F(x)$.

56. $G(x) = \begin{cases} -x + 3, & \text{for } x < 2, \\ x + 1, & \text{for } x \geq 2. \end{cases}$

Find $\lim_{x \to 2^-} G(x)$, $\lim_{x \to 2^+} G(x)$, and $\lim_{x \to 2} G(x)$.

57. $g(x) = \begin{cases} -x + 4, & \text{for } x < 3, \\ x - 3, & \text{for } x > 3. \end{cases}$

Find $\lim_{x \to 3^-} g(x)$, $\lim_{x \to 3^+} g(x)$, and $\lim_{x \to 3} g(x)$.

58. $f(x) = \begin{cases} 3x - 4, & \text{for } x < 1, \\ x - 2, & \text{for } x > 1. \end{cases}$

Find $\lim_{x \to 1^-} f(x)$, $\lim_{x \to 1^+} f(x)$, and $\lim_{x \to 1} f(x)$.

59. $G(x) = \begin{cases} x^2, & \text{for } x < -1, \\ x + 2, & \text{for } x > -1. \end{cases}$ Find $\lim_{x \to -1} G(x)$.

60. $F(x) = \begin{cases} -2x - 3, & \text{for } x < -1, \\ x^3, & \text{for } x > -1. \end{cases}$ Find $\lim_{x \to -1} F(x)$.

APPLICATIONS

Business and Economics

Taxicab fares. *In New York City, taxicabs charge passengers \$2.50 for entering a cab and then \$0.40 for each one-fifth of a mile (or fraction thereof) traveled. (There are additional charges for slow traffic and idle times, but these are not considered in this problem.) If x represents the distance traveled in miles, then C(x) is the cost of the taxi fare, where*

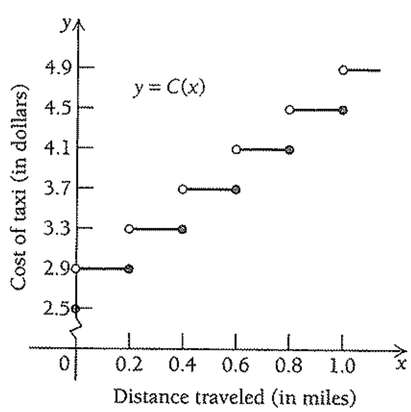
$C(x) = \$2.50$, if $x = 0$,

$C(x) = \$2.90$, if $0 < x \le 0.2$,

$C(x) = \$3.30$, if $0.2 < x \le 0.4$,

$C(x) = \$3.70$, if $0.4 < x \le 0.6$,

and so on. The graph of C is shown below. (Source: New York City Taxi and Limousine Commission.)



Using the graph of the taxicab fare function, find each of the following limits, if it exists.

61. $\lim_{x \to 0.25^-} C(x)$, $\lim_{x \to 0.25^+} C(x)$, $\lim_{x \to 0.25} C(x)$

62. $\lim_{x \to 0.2^-} C(x)$, $\lim_{x \to 0.2^+} C(x)$, $\lim_{x \to 0.2} C(x)$

63. $\lim_{x \to 0.6^-} C(x)$, $\lim_{x \to 0.6^+} C(x)$, $\lim_{x \to 0.6} C(x)$

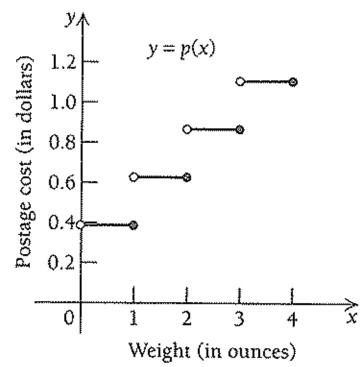
The postage function. *Postal rates are \$0.39 for the first ounce and \$0.24 for each additional ounce (or fraction thereof). If x represents the weight of a letter in ounces, then p(x) is the cost of mailing the letter, where*

$p(x) = \$0.39$, if $0 < x \le 1$,

$p(x) = \$0.63$, if $1 < x \le 2$,

$p(x) = \$0.87$, if $2 < x \le 3$,

and so on, up to 13 ounces (at which point postage cost also depends on distance). The graph of p is shown below.



Using the graph of the postage function, find each of the following limits, if it exists.

64. $\lim_{x \to 1^-} p(x)$, $\lim_{x \to 1^+} p(x)$, $\lim_{x \to 1} p(x)$

65. $\lim_{x \to 2^-} p(x)$, $\lim_{x \to 2^+} p(x)$, $\lim_{x \to 2} p(x)$

66. $\lim_{x \to 2.6^-} p(x)$, $\lim_{x \to 2.6^+} p(x)$, $\lim_{x \to 2.6} p(x)$

67. $\lim_{x \to 3} p(x)$

68. $\lim_{x \to 3.4} p(x)$

Natural Sciences

Population growth. *In a certain habitat, the deer population (in hundreds) as a function of time (in years) is given in the graph of p below.*