Exercise Set 1.5

Find $\frac{dy}{dx}$.

1. $y = x^7$
2. $y = x^8$
3. $y = -3x$
4. $y = -0.5x$
5. $y = 12$
6. $y = 7$
7. $y = 2x^{15}$
8. $y = 3x^{10}$
9. $y = x^{-6}$
10. $y = x^{-8}$
11. $y = 4x^{-2}$
12. $y = 3x^{-5}$
13. $y = x^3 + 3x^2$
14. $y = x^4 - 7x$
15. $y = 8\sqrt{x}$
16. $y = 4\sqrt{x}$
17. $y = x^{0.9}$
18. $y = x^{0.7}$
19. $y = \frac{1}{2}x^{4/5}$
20. $y = -4.8x^{1/3}$
21. $y = \frac{7}{x^3}$
22. $y = \frac{6}{x^4}$
23. $y = \frac{4x}{5}$
24. $y = \frac{3x}{4}$

Find each derivative.

25. $\frac{d}{dx}\left(\sqrt[4]{x} - \frac{3}{x}\right)$
26. $\frac{d}{dx}\left(\sqrt[5]{x} - \frac{2}{x}\right)$
27. $\frac{d}{dx}\left(\sqrt{x} - \frac{2}{\sqrt{x}}\right)$
28. $\frac{d}{dx}\left(\sqrt[3]{x} + \frac{4}{\sqrt{x}}\right)$
29. $\frac{d}{dx}\left(-2\sqrt[3]{x^5}\right)$
30. $\frac{d}{dx}\left(-\sqrt[4]{x^3}\right)$
31. $\frac{d}{dx}(5x^2 - 7x + 3)$
32. $\frac{d}{dx}(6x^2 - 5x + 9)$

Find $f'(x)$.

33. $f(x) = 0.6x^{1.5}$
34. $f(x) = 0.3x^{1.2}$
35. $f(x) = \frac{2x}{3}$
36. $f(x) = \frac{3x}{4}$
37. $f(x) = \frac{4}{7x^3}$
38. $f(x) = \frac{2}{5x^6}$
39. $f(x) = \frac{5}{x} - x^{2/3}$
40. $f(x) = \frac{4}{x} - x^{3/5}$
41. $f(x) = 4x - 7$
42. $f(x) = 7x - 14$
43. $f(x) = \frac{x^{4/3}}{4}$
44. $f(x) = \frac{x^{3/2}}{3}$
45. $f(x) = -0.01x^2 - 0.5x + 70$
46. $f(x) = -0.01x^2 + 0.4x + 50$

Find y'.

47. $y = 3x^{-2/3} + x^{3/4} + x^{6/5} + \frac{8}{x^3}$
48. $y = x^{-3/4} - 3x^{2/3} + x^{5/4} + \frac{2}{x^4}$
49. $y = \frac{2}{x} - \frac{x}{2}$
50. $y = \frac{x}{7} + \frac{7}{x}$
51. If $f(x) = x^2 + 4x - 5$, find $f'(10)$.
52. If $f(x) = \sqrt{x}$, find $f'(4)$.
53. If $y = \frac{4}{x^2}$, find $\left.\frac{dy}{dx}\right|_{x=-2}$
54. If $y = x + \frac{2}{x^3}$, find $\left.\frac{dy}{dx}\right|_{x=1}$
55. Find an equation of the tangent line to the graph of $f(x) = x^3 - 2x + 1$
   a) at $(2, 5)$; **b)** at $(-1, 2)$; **c)** at $(0, 1)$.
56. Find an equation of the tangent line to the graph of $f(x) = x^2 - \sqrt{x}$
   a) at $(1, 0)$; **b)** at $(4, 14)$; **c)** at $(9, 78)$.

For each function, find the points on the graph at which the tangent line is horizontal. If none exist, state that fact.

57. $y = x^2 - 3$
58. $y = -x^2 + 4$
59. $y = -x^3 + 1$
60. $y = x^3 - 2$
61. $y = 3x^2 - 5x + 4$
62. $y = 5x^2 - 3x + 8$
63. $y = -0.01x^2 - 0.5x + 70$
64. $y = -0.01x^2 + 0.4x + 50$
65. $y = 2x + 4$
66. $y = -2x + 5$
67. $y = 4$
68. $y = -3$
69. $y = -x^3 + x^2 + 5x - 1$
70. $y = -\frac{1}{3}x^3 + 6x^2 - 11x - 50$
71. $y = \frac{1}{3}x^3 - 3x + 2$
72. $y = x^3 - 6x + 1$
73. $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2$
74. $f(x) = \frac{1}{3}x^3 - 3x^2 + 9x - 9$

For each function, find the points on the graph at which the tangent line has slope 1.

75. $y = 20x - x^2$
76. $y = 6x - x^2$
77. $y = -0.025x^2 + 4x$
78. $y = -0.01x^2 + 2x$
79. $y = \frac{1}{3}x^3 + 2x^2 + 2x$
80. $y = \frac{1}{3}x^3 - x^2 - 4x + 1$

APPLICATIONS

Life and Natural Sciences

81. **Healing wound.** The circular area A, in square centimeters, of a healing wound is approximated by

$$A(r) = 3.14r^2,$$

where r is the wound's radius, in centimeters.

a) Find the rate of change of the area with respect to the radius.

TW b) Explain the meaning of your answer to part (a).

82. **Healing wound.** The circumference C, in centimeters, of a healing wound is approximated by

$$C(r) = 6.28r,$$

where r is the wound's radius, in centimeters.

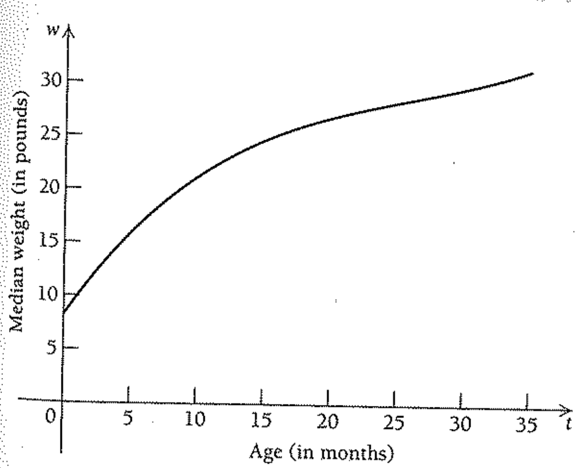
a) Find the rate of change of the circumference with respect to the radius.

TW b) Explain the meaning of your answer to part (a).

83. **Growth of a baby.** The median weight of a boy whose age is between 0 and 36 months can be approximated by the function

$$w(t) = 8.15 + 1.82t - 0.0596t^2 + 0.000758t^3,$$

where t is measured in months and w is measured in pounds.



(*Source*: Centers for Disease Control. Developed by the National Center for Health Statistics in collaboration with the National Center for Chronic Disease Prevention and Health Promotion, 2000.)

Use this approximation to find the following for a boy with median weight:

a) The rate of change of weight with respect to time.

b) The weight of the baby at age 10 months.

c) The rate of change of the baby's weight with respect to time at age 10 months.

84. **Temperature during an illness.** The temperature T of a person during an illness is given by

$$T(t) = -0.1t^2 + 1.2t + 98.6,$$

where T is the temperature, in degrees Fahrenheit, at time t, in days.

a) Find the rate of change of the temperature with respect to time.

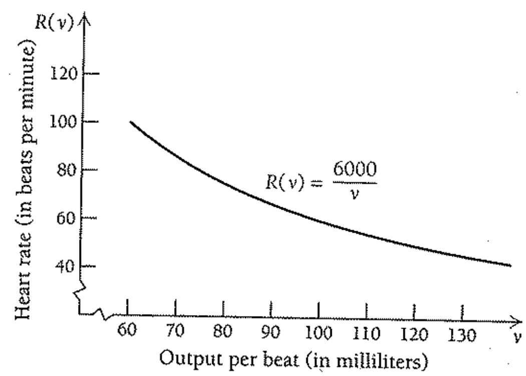
b) Find the temperature at $t = 1.5$ days.

c) Find the rate of change at $t = 1.5$ days.

85. **Heart rate.** The equation

$$R(v) = \frac{6000}{v}$$

can be used to determine the heart rate, R, of a person whose heart pumps 6000 milliliters (mL) of blood per minute and v milliliters of blood per beat. (*Source*: *Mathematics Teacher*, Vol. 99, No. 4, November 2005.)



a) Find the rate of change of heart rate with respect to v, the output per beat.

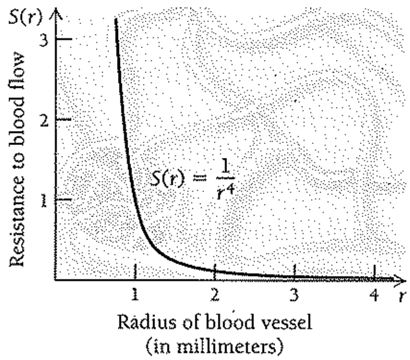
b) Find the heart rate at $v = 80$ mL per beat.

c) Find the rate of change at $v = 80$ mL per beat.

86. **Blood flow resistance.** The equation

$$S(r) = \frac{1}{r^4}$$

can be used to determine the resistance to blood flow, S, of a blood vessel that has radius r, in millimeters (mm). (*Source: Mathematics Teacher*, Vol. 99, No. 4, November 2005.)



a) Find the rate of change of resistance with respect to r, the radius of the blood vessel.
b) Find the resistance at $r = 1.2$ mm.
c) Find the rate of change of S with respect to r when $r = 0.8$ mm.

Social Sciences

87. Population growth rate. The population of a city grows from an initial size of 100,000 to a size P given by

$$P(t) = 100{,}000 + 2000t^2,$$

where t is in years.

a) Find the **growth rate,** dP/dt.
b) Find the population after 10 yr.
c) Find the growth rate at $t = 10$.
TW **d)** Explain the meaning of your answer to part (c).

88. Median age of women at first marriage. The median age of women at first marriage can be approximated by the linear function

$$A(t) = 0.08t + 19.7,$$

where $A(t)$ is the median age of women marrying for the first time at t years after 1950.

a) Find the rate of change of the median age A with respect to time t.
TW **b)** Explain the meaning of your answer to part (a).

General Interest

89. View to the horizon. The view V, or distance in miles, that one can see to the horizon from a height h, in feet, is given by

$$V = 1.22\sqrt{h}.$$

a) Find the rate of change of V with respect to h.
b) How far can one see to the horizon from an airplane window at a height of 40,000 ft?
c) Find the rate of change at $h = 40{,}000$.
TW **d)** Explain the meaning of your answers to parts (a) and (c).

90. Baseball ticket prices. The average price, in dollars, of a ticket for a Major League baseball game x years after 1990 can be estimated by

$$p(x) = 9.41 - 0.19x + 0.09x^2.$$

a) Find the rate of change of the average ticket price with respect to the year, dp/dx.
b) What is the average ticket price in 2007?
c) What is the rate of change of the average ticket price in 2007?

SYNTHESIS

For each function, find the interval(s) for which $f'(x)$ is positive.

91. $f(x) = x^2 - 4x + 1$

92. $f(x) = x^2 + 7x + 2$

93. $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 5$

94. Find the points on the graph of
$$y = x^4 - \tfrac{4}{3}x^2 - 4$$
at which the tangent line is horizontal.

95. Find the points on the graph of
$$y = 2x^6 - x^4 - 2$$
at which the tangent line is horizontal.

Find dy/dx. Each function can be differentiated using the rules developed in this section, but some algebra may be required beforehand.

96. $y = (x + 3)(x - 2)$

97. $y = (x - 1)(x + 1)$

98. $y = \dfrac{x^5 - x^3}{x^2}$

99. $y = \dfrac{5x^2 - 8x + 3}{8}$

100. $y = \dfrac{x^5 + x}{x^2}$

101. $y = \dfrac{x^5 - 3x^4 + 2x + 4}{x^2}$

102. $y = (-4x)^3$

103. $y = \sqrt{7x}$

104. $y = \sqrt[3]{8x}$

105. $y = (x - 3)^2$

106. $y = \left(\sqrt{x} - \dfrac{1}{\sqrt{x}}\right)^2$

107. $y = (\sqrt{x} + \sqrt[3]{x})^2$

108. $y = (x + 1)^3$

109. Use Theorem 1 to prove that the derivative of 1 is 0.

Exercise Set 1.6

Differentiate two ways: first, by using the Product Rule; then, by multiplying the expressions before differentiating. Compare your results as a check.

1. $y = x^5 \cdot x^6$

2. $y = x^9 \cdot x^4$

3. $f(x) = (2x + 5)(3x - 4)$

4. $g(x) = (3x - 2)(4x + 1)$

5. $G(x) = 4x^2(x^3 + 5x)$

6. $F(x) = 3x^4(x^2 - 4x)$

7. $y = (3\sqrt{x} + 2)x^2$

8. $y = (4\sqrt{x} + 3)x^3$

9. $g(x) = (4x - 3)(2x^2 + 3x + 5)$

10. $f(x) = (2x + 5)(3x^2 - 4x + 1)$

11. $F(t) = (\sqrt{t} + 2)(3t - 4\sqrt{t} + 7)$

12. $G(t) = (2t + 3\sqrt{t} + 5)(\sqrt{t} + 4)$

Differentiate two ways: first, by using the Quotient Rule; then, by dividing the expressions before differentiating. Compare your results as a check.

13. $y = \dfrac{x^7}{x^3}$

14. $y = \dfrac{x^6}{x^4}$

15. $f(x) = \dfrac{2x^5 + x^2}{x}$

16. $g(x) = \dfrac{3x^7 - x^3}{x}$

17. $G(x) = \dfrac{8x^3 - 1}{2x - 1}$

18. $F(x) = \dfrac{x^3 + 27}{x + 3}$

19. $y = \dfrac{t^2 - 16}{t + 4}$

20. $y = \dfrac{t^2 - 25}{t - 5}$

Differentiate each function.

21. $f(x) = (3x^2 - 2x + 5)(4x^2 + 3x - 1)$

22. $g(x) = (5x^2 + 4x - 3)(2x^2 - 3x + 1)$

23. $y = \dfrac{5x^2 - 1}{2x^3 + 3}$

24. $y = \dfrac{3x^4 + 2x}{x^3 - 1}$

25. $G(x) = (8x + \sqrt{x})(5x^2 + 3)$

26. $F(x) = (-3x^2 + 4x)(7\sqrt{x} + 1)$

27. $g(t) = \dfrac{t}{3 - t} + 5t^3$

28. $f(t) = \dfrac{t}{5 + 2t} - 2t^4$

29. $F(x) = (x + 3)^2$
[*Hint:* $(x + 3)^2 = (x + 3)(x + 3)$.]

30. $G(x) = (5x - 4)^2$

31. $y = (x^3 - 4x)^2$

32. $y = (3x^2 - 4x + 5)^2$

33. $g(x) = 5x^{-3}(x^4 - 5x^3 + 10x - 2)$

34. $f(x) = 6x^{-4}(6x^3 + 10x^2 - 8x + 3)$

35. $F(t) = \left(t + \dfrac{2}{t}\right)(t^2 - 3)$

36. $G(t) = (3t^5 - t^2)\left(t - \dfrac{5}{t}\right)$

37. $y = \dfrac{x^2 + 1}{x^3 - 1} - 5x^2$

38. $y = \dfrac{x^3 - 1}{x^2 + 1} + 4x^3$

39. $y = \dfrac{\sqrt[3]{x} - 7}{\sqrt{x} + 3}$

40. $y = \dfrac{\sqrt{x} + 4}{\sqrt[3]{x} - 5}$

41. $f(x) = \dfrac{x}{x^{-1} + 1}$

42. $f(x) = \dfrac{x^{-1}}{x + x^{-1}}$

43. $F(t) = \dfrac{1}{t - 4}$

44. $G(t) = \dfrac{1}{t + 2}$

45. $f(x) = \dfrac{3x^2 + 2x}{x^2 + 1}$

46. $f(x) = \dfrac{3x^2 - 5x}{x^2 - 1}$

47. $g(t) = \dfrac{-t^2 + 3t + 5}{t^2 - 2t + 4}$

48. $f(t) = \dfrac{3t^2 + 2t - 1}{-t^2 + 4t + 1}$

49–96. Use a graphing calculator to check the results of Exercises 1–48.

97. Find an equation of the tangent line to the graph of $y = 8/(x^2 + 4)$ at **(a)** $(0, 2)$; **(b)** $(-2, 1)$.

98. Find an equation of the tangent line to the graph of $y = \sqrt{x}/(x + 1)$ at **(a)** $x = 1$; **(b)** $x = \frac{1}{4}$.

99. Find an equation of the tangent line to the graph of $y = x^2 + 3/(x - 1)$ at **(a)** $x = 2$; **(b)** $x = 3$.

100. Find an equation of the tangent line to the graph of $y = 4x/(1 + x^2)$ at **(a)** $(0, 0)$; **(b)** $(-1, -2)$.

APPLICATIONS

Business and Economics

101. Average cost. Summertime Fabrics finds that the cost, in dollars, of producing x jackets is given by $C(x) = 950 + 15\sqrt{x}$. Find the rate at which the average cost is changing when 400 jackets have been produced.

102. Average cost. Tongue-Tied Sauces, Inc., finds that the cost, in dollars, of producing x bottles of barbecue sauce is given by $C(x) = 375 + 0.75x^{3/4}$. Find the rate at which the average cost is changing when 81 bottles of barbecue sauce have been produced.

103. Average revenue. Summertime Fabrics finds that the revenue, in dollars, from the sale of x jackets is given by $R(x) = 85\sqrt{x}$. Find the rate at which average revenue is changing when 400 jackets have been produced.

104. Average revenue. Tongue-Tied Sauces, Inc., finds that the revenue, in dollars, from the sale of x bottles of barbecue sauce is given by $R(x) = 7.5x^{0.7}$. Find the rate at which average revenue is changing when 81 bottles of barbecue sauce have been produced.

105. Average profit. Use the information in Exercises 101 and 103 to determine the rate at which Summertime Fabrics' average profit per jacket is changing when 400 jackets have been produced and sold.

106. Average profit. Use the information in Exercises 102 and 104 to determine the rate at which Tongue-Tied Sauces' average profit per bottle of barbecue sauce is changing when 81 bottles have been produced and sold.

107. Average profit. Sparkle Pottery has determined that the cost, in dollars, of producing x vases is given by

$$C(x) = 4300 + 2.1x^{0.6}.$$

If the revenue from the sale of x vases is given by $R(x) = 65x^{0.9}$, find the rate at which the average profit per vase is changing when 50 vases have been made and sold.

108. Average profit. Cruzin' Boards has found that the cost, in dollars, of producing x skateboards is given by

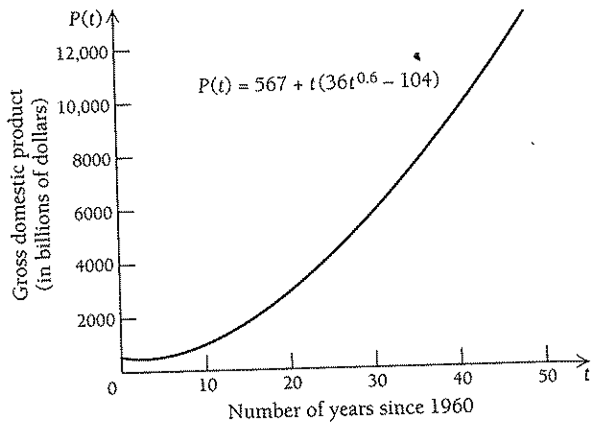
$$C(x) = 900 + 18x^{0.7}.$$

If the revenue from the sale of x skateboards is given by $R(x) = 75x^{0.8}$, find the rate at which the average profit per skateboard is changing when 20 skateboards have been built and sold.

109. Gross domestic product. The U.S. gross domestic product (in billions of dollars) can be approximated using the function

$$P(t) = 567 + t(36t^{0.6} - 104),$$

where t is the number of the years since 1960.



(*Source*: U.S. Bureau of Economic Analysis.)

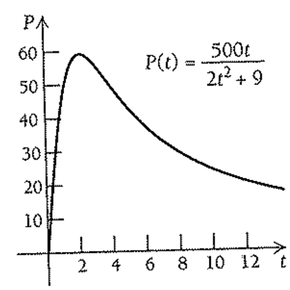
a) Find $P'(t)$.
b) Find $P'(45)$.
TW c) In words, explain what $P'(45)$ means.

Social Sciences

110. Population growth. The population P, in thousands, of a small city is given by

$$P(t) = \frac{500t}{2t^2 + 9},$$

where t is the time, in months.



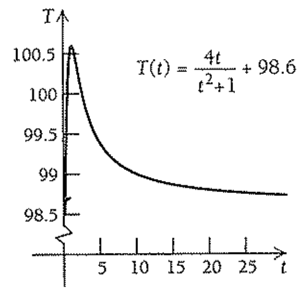
a) Find the growth rate.
b) Find the population after 12 months.
c) Find the growth rate at $t = 12$ months.

Life and Physical Sciences

111. Temperature during an illness. The temperature T of a person during an illness is given by

$$T(t) = \frac{4t}{t^2 + 1} + 98.6,$$

where T is the temperature, in degrees Fahrenheit, at time t, in hours.



a) Find the rate of change of the temperature with respect to time.
b) Find the temperature at $t = 2$ hr.
c) Find the rate of change of the temperature at $t = 2$ hr.

SYNTHESIS

Differentiate each function.

112. $f(x) = \dfrac{7 - \dfrac{3}{2x}}{\dfrac{4}{x^2} + 5}$ (*Hint:* Simplify before differentiating.)

113. $y(t) = 5t(t - 1)(2t + 3)$

114. $f(x) = x(3x^3 + 6x - 2)(3x^4 + 7)$

115. $g(x) = (x^3 - 8) \cdot \dfrac{x^2 + 1}{x^2 - 1}$

116. $f(t) = (t^5 + 3) \cdot \dfrac{t^3 - 1}{t^3 + 1}$

117. $f(x) = \dfrac{(x - 1)(x^2 + x + 1)}{x^4 - 3x^3 - 5}$

118. Let $f(x) = \dfrac{x}{x + 1}$ and $g(x) = \dfrac{-1}{x + 1}$.

a) Compute $f'(x)$.
b) Compute $g'(x)$.
TW c) What can you conclude about f and g on the basis of your results from parts (a) and (b)?

119. Let $f(x) = \dfrac{x^2}{x^2 - 1}$ and $g(x) = \dfrac{1}{x^2 - 1}$.

a) Compute $f'(x)$.
b) Compute $g'(x)$.
TW c) What can you conclude about the graphs of f and g on the basis of your results from parts (a) and (b)?

TW **120.** Write a rule for finding the derivative of $f(x) \cdot g(x) \cdot h(x)$. Describe the rule in words.

TW **121.** Is the derivative of the reciprocal of $f(x)$ the reciprocal of the derivative of $f'(x)$? Why or why not?

122. Sensitivity. The reaction R of the body to a dose Q of medication is often represented by the general function

$$R(Q) = Q^2\left(\frac{k}{2} - \frac{Q}{3}\right),$$

where k is a constant and R is in millimeters of mercury (mmHg) if the reaction is a change in blood pressure or in degrees Fahrenheit (°F) if the reaction is a change in temperature. The rate of change dR/dQ is defined to be the body's *sensitivity* to the medication.

a) Find a formula for the sensitivity.
TW b) Explain the meaning of your answer to part (a).

123. A proof of the Product Rule appears below. Provide a justification for each step.

a) $\dfrac{d}{dx}[f(x) \cdot g(x)] = \lim\limits_{h \to 0} \dfrac{f(x + h)g(x + h) - f(x)g(x)}{h}$

b) $= \lim\limits_{h \to 0} \dfrac{f(x + h)g(x + h) - f(x + h)g(x) + f(x + h)g(x) - f(x)g(x)}{h}$

c) $= \lim\limits_{h \to 0} \dfrac{f(x + h)g(x + h) - f(x + h)g(x)}{h} + \lim\limits_{h \to 0} \dfrac{f(x + h)g(x) - f(x)g(x)}{h}$

d) $= \lim\limits_{h \to 0} \left[f(x + h) \cdot \dfrac{g(x + h) - g(x)}{h}\right] + \lim\limits_{h \to 0} \left[g(x) \cdot \dfrac{f(x + h) - f(x)}{h}\right]$

e) $= f(x) \cdot \lim\limits_{h \to 0} \dfrac{g(x + h) - g(x)}{h} + g(x) \cdot \lim\limits_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$

f) $= f(x) \cdot g'(x) + g(x) \cdot f'(x)$

g) $= f(x) \cdot \left[\dfrac{d}{dx}g(x)\right] + g(x) \cdot \left[\dfrac{d}{dx}f(x)\right]$