Key

Name (L.F): $\cdots\ \cdots$ Student ID: $\cdots\ \cdots$

TEST 1 (1710-FALL,012)

INSTRUCTOR: KOSHAL DAHAL

There are 12 questions plus 4 extra credit questions. Each question is worth 8 points, except extra credit questions which is worth 4 points each. You get 4 points for putting your name (only if it matches to the official university roster) on the paper, which adds up to 100 (116 with extra credit).

Please box your answers. Show all relevant work. Make sure that no notes, no books, no extra paper, no caps/hats or any electronic devices are allowed to be used, but only pencil, eraser and your own. Please Turn-off your cell phones!

Q1: Let

$$f(x) = \begin{cases} x^2+1 & x < -1 \\ 0 & x = -1 \\ \sqrt{x+2} & x > -1 \end{cases}$$

Compute the following limits, or state that they do not exist.

(A) $\lim_{x\to -1^-} f(x) = 2$

(B) $\lim_{x\to -1^+} f(x) = 1$

(C) $\lim_{x\to -1} f(x)$ DNE as $2 \neq 1$.

Q2: Find the following limits:

(A) $\lim_{z\to 4} \frac{z-5}{(z^2-10z+24)^2} = \frac{(z-5)}{[(z-4)(z-6)]^2} = \frac{z(1-5/z)}{z^4[(1-4/z)(1-6/z)]^2}$

take limit as $z \to 4$ gives $= -\infty$.

(B) $\lim_{x\to -\infty} \frac{\cos x^5}{x} = 0$ by squeeze thrm (as $-1/x \le \frac{\cos x^5}{x} \le 1/x$)..

Q.3: Let $f(x) = \frac{2x}{\sqrt{x^2-x-2}}$, then answer the followings

$= \frac{2}{\sqrt{1-1/x-2/x^2}}$

(A) Find $\lim_{x\to+\infty} f(x)$ = 2

(B) Find $\lim_{x\to-\infty} f(x)$ = 2

$\Longrightarrow$ Horiz. asymptote $y=2$

put $x^2-x-2=0$ ∴ $x=2$, $x=-1$

(C) Find the vertical and horizontal asymptotes

Q.4: Is f(x) defined by

$$f(x) = \begin{cases} \frac{1}{x-3} & x > 2 \\ 2 & 1 < x \leq 2 \\ x+1 & x \leq 1 \end{cases}$$

left continuous, right continuous or continuous at 1, 2, and 3? Prove your answer explicitly.

- Continuous @ 1.
- only left continuous @ 2.
- Not continuous @ 3 as f(3) DNE.

Q.5: (A) Give any three (3) examples of a continuous function. If f is continuous at a, must f be differentiable at a? Explain why or give a counterexample.

- Trig. functions, polynomials, Abs. value functions, . . .
- No, ex. $f(x) = |x|$ at 0.

(B) The equation $x^3 - 5x^2 + 2x = -1$ has at most three solutions. Show that, using IVT, it has a solution on the given interval $[-1, 5]$.

$f(-1) = -7 < 0$, $f(5) = 11 > 0$.

by IVT ∃ a sol^n of f in [-1,5] as f changes sign.

Q.6: Find the equation of the line tangent to the graph $y = x^3 - 4x^2 + 2x - 1$ at the point $(2, -5)$.

$y' = 3x^2 - 8x + 2$

@ $(2, -5)$, $m = y'\big|_{x=2} = -2$

$\therefore$ Eqⁿ of tangent $y + 5 = -2(x - 2)$

$\therefore y = -2x - 1.$

Q.7: Suppose $s = f(t)$ is the position of a particle at time t.

(A) What does $f'(t)$ represent?

velocity

(B) What does $f''(t)$ represent?

Acceleration

(C) Suppose the average cost of producing $x = 555$ toy-iphones is \$5.55 per toy-iphone and the marginal cost at $x = 555$ is \$4.55 per toy-iphone. Interpret these costs.

- on average to produce each of first 555 iphones, it costs \$5.55. and if 555 have already been produced then the next one (the 556th) costs \$4.55 to produce.

In problems 8-9, find $\frac{dy}{dx}$ [hint: $(N/D)' = \frac{D.N' - N.D'}{D^2}$]

Q.8: $y = \frac{(x-1)(2x^2-1)}{(x^3-1)}$ $= \dfrac{2x^2 - 1}{x^2 + x + 1}$

$$y' = \frac{(x^2+x+1)4x - (2x^2-1)(2x+1)}{(x^2+x+1)^2} = \frac{2x^2+6x+1}{(x^2+x+1)^2}.$$

Q.9: $y = \frac{(x^2-1)\sin x}{\sin x+1}$

$$y' = \frac{(\sin x+1)\left[(2x\cdot\sin x + (x^2-1)\cos x\right] - (x^2-1)\sin x\cdot\cos x}{(\sin x+1)^2}$$

$$= \frac{2x\sin^2 x + 2x\cdot\sin x + x^2\cdot\cos x - \cos x}{(\sin x+1)^2}.$$

Q.10: Find $\frac{d^3y}{dx^3}$, where $y = x^2(2 + x^{-3})$

$$= 2x^2 + 1/x$$

$$y' = 4x - 1/x^2$$

$$y'' = 4 + 2/x^3$$

$$y''' = -6/x^4 = -6\cdot x^{-4}.$$

Q.11:(A) The following limit equal the derivative of a function f at a point a. Evaluate the limit

$$\lim_{x\to\frac{\pi}{4}} \frac{\cot x - 1}{x - \frac{\pi}{4}}$$ let $f(x) = \cot x$, $a = \pi/4$.

$$= f'(\pi/4) = -\csc^2 x\Big|_{x=\pi/4} = -2.$$

(B) During your trip to UNT, your car will undergo a series of changes in its speed. Then what does the speedometer of your car measure: average speed or the instantaneous speed? (Note that speed = $|velocity|$)

Q.12: State the three conditions for the function $f(x)$ NOT to be differentiable at a point $x = a$. — if f has Corner at a.

" Vert. asymptote at a.

" is not Continuous at a.

Q.13: (Extra Credit) Does there exist a real number x such that cos(x) = x? Prove your answer.

let $f(x) = \cos x - x$.

Here $f(0) = 1 > 0$ & $f(\pi) = -1 - \pi < 0$

use IVT. to guarantee a real x s.t. $f(x) = 0$

$\Rightarrow \cos x = x$.

Q.14: (Extra Credit) Evaluate the following limit: $\lim_{x \to 0} x^2 \, cos(\frac{1}{x})$ $= 0$

by squeeze thrm.

Since $-1 \le \cos(1/x) \le 1$ & then

multiply by x^2 throughout.

Q.15: (Extra Credit) Determine whether the following statements are true and give an explanation or counterexample.

(A) If a child's temperature rose from 98.8^o to 102.3^o, then can you conclude that there was an instant when the child's temperature was exactly 100^o.

True, ($\because$ IVT)

(B) In 1987 it cost \$.22 to mail a letter frist class inside the United States, and in 1990 it cost \$.25 to mail the same letter, then can you conclude that there was a time when it cost \$.24 to send the letter. False ($\because$ postal price doesn't obey continuity, so not applicable IVT).

Q.16: (Extra Credit) Fill in the blanks with the appropriate given choices: can, can't, less than or equal, greater, more/less, positive, negative, zero, increasing, decreasing.

A graph can $\cdots$ cross a horizontal asymptote; can $\cdots$ cross a slant asymptote; can't $\cdots$ cross a vertical asymptote. The horizontal asymptote occurs when the degree of the numerator is $\le$ $\cdots$ to the degree of the denominator. So no horizontal asymptote exists if the degree of numerator is $>$ $\cdots$ than the degree of denominator. A slant asymptote occurs when the degree of the numerator is exactly 1 more $\cdots$ than the degree of the denominator.

Likewise, if the slope of the tangent line is positive, then f' is +ve $\cdots$; if the slope of the tangent line is negative, then f' is -ve $\cdots$; if the tangent line is horizontal, then f' is $\cdot$ 0. This leads to the statements like: If $f' > 0$ on an interval, then we say f is $\cdot$ $\nearrow$ over that interval; if $f' < 0$ on an interval, then we say f is $\cdots$ $\searrow$ over that interval.