KEY

Test 3 (Cal 1) Fall 2012

Date: 12/03/2012

INSTRUCTOR:- KOSHAL DAHAL

Name (L,F) : ______________________

MATH

ID/EuID: 1710-007

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Express the sum in sigma notation.

1) $4 + 8 + 12 + 16 + 20$ 1) ______

A) $\sum_{k=1}^{6} 4k$ B) $\sum_{k=2}^{5} 4(k-1)$ C) $\sum_{k=1}^{5} 4(k+1)$ D) $\sum_{k=0}^{4} 4(k+1)$

Express the sum in sigma notation.

2) $1 - 4 + 16 - 64 + 256$ 2) ______

A) $\sum_{k=-1}^{3} (-1)^{k+1} 4^k$

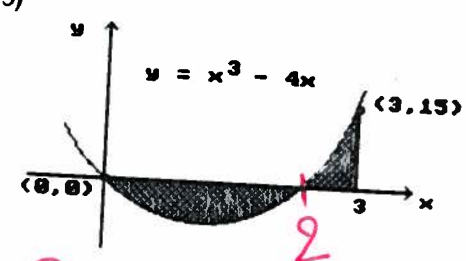
B) $\sum_{k=-2}^{2} (-1)^{k+1} 4^{k+1}$

C) $\sum_{k=1}^{5} (-4)^k$

D) $\sum_{k=0}^{4} (-1)^k 4^k$

Find the area of the shaded region.

3) 3) ______



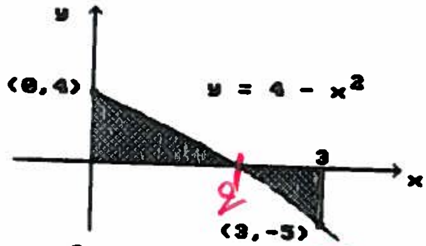
$$\int_0^2 -(x^3-4x)\,dx + \int_2^3 (x^3-4x)\,dx$$

$$= -\left(\frac{x^4}{4} - 2x^2\Big|_0^2\right) + \frac{x^4}{4} - 2x^2\Big|_2^3$$

A) $\frac{41}{4}$ B) $\frac{33}{4}$ C) $\frac{17}{4}$ D) $\frac{9}{4}$

$$= \frac{16}{4} + \frac{25}{4} = 41/4.$$

4)



4) ______

A) $\frac{5}{3}$ B) $\frac{23}{3}$ C) 3 D) 5

$$\int_0^2 (4-x^2)\,dx + \left(-\int_2^3 (4-x^2)\,dx\right)$$

$$= 4x - \frac{x^3}{3}\Big|_0^2 - \left(4x - \frac{x^3}{3}\right)\Big|_2^3$$

$$= \frac{16}{3} + \frac{7}{3} = \frac{23}{3}$$

Find the total area of the region between the curve and the x-axis.

5) $y = x^2(x - 2)^2;\ 0 \le x \le 2$

5) ______

A) $\frac{15}{16}$ B) $\frac{17}{15}$ C) $\frac{15}{17}$ D) $\frac{16}{15}$

$$\int_0^2 x^2 (x-2)^2\,dx = \int_0^2 x^2 (x^2 - 4x + 4)\,dx$$

$$= \int_0^2 (x^4 - 4x^3 + 4x^2)\,dx$$

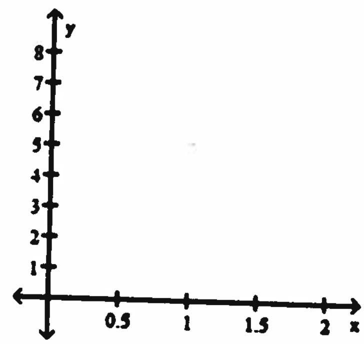
$$= \frac{x^5}{5} - x^4 + \frac{4}{3}x^3 \Big|_0^2$$

$$= \frac{16}{15}$$

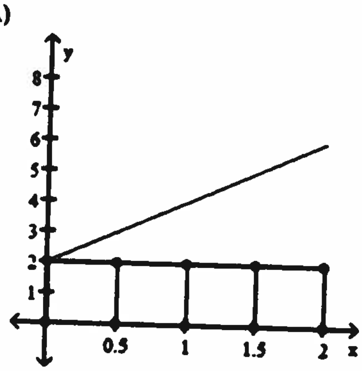
Graph the function $f(x)$ over the given interval. Partition the interval into 4 subintervals of equal length. Then add to your sketch the rectangles associated with the Riemann sum $\sum_{k=1}^{4} f(c_k)\,\Delta x_k$, using the indicated point in the kth subinterval for c_k.

6) $f(x) = 2x + 2$, $[0, 2]$, left-hand endpoint

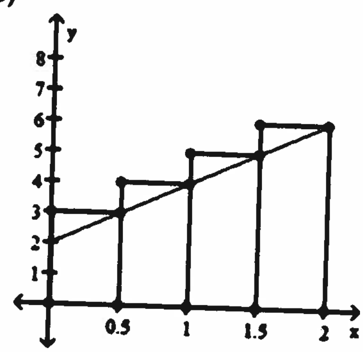
6) ______



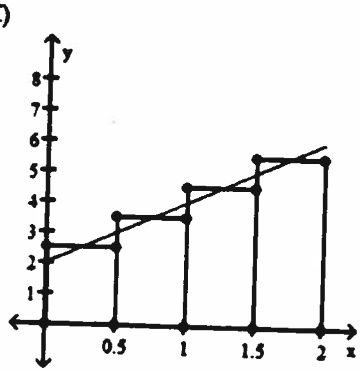
A)



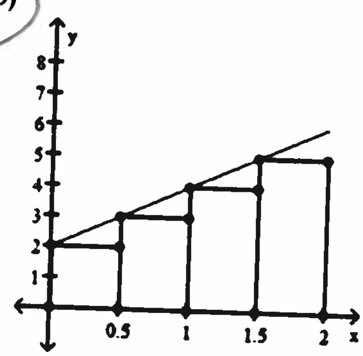
B)



C)



D)



Find the derivative.

7) $y = \int_0^x \sqrt{6t + 3}\, dt$ 7) ______

A) $\frac{1}{9}(6x + 3)^{3/2}$ B) $\frac{3}{\sqrt{6x + 3}}$ C) $\sqrt{6x + 3} - \sqrt{3}$ D) $\sqrt{6x + 3}$

8) $y = \int_0^{\tan x} \sqrt{t}\, dt$ 8) ______

A) $\sec^2 x \sqrt{\tan x}$ B) $\frac{2}{3} \tan^{3/2} x$ C) $\sec x \tan^{3/2} x$ D) $\sqrt{\tan x}$

$\sqrt{\tan x} \cdot \sec^2 x.$

Evaluate the integral.

9) $\int_1^4 \frac{t^2 + 1}{\sqrt{t}}\, dt$ $= \int_1^4 \left(t^{3/2} + t^{-1/2}\right) dt = \frac{2}{5} t^{5/2} + 2t^{1/2} \Big|_1^4 = \frac{72}{5}$ 9) ______

A) $\frac{92}{5}$ B) $\frac{72}{5}$ C) $\frac{77}{5}$ D) 32

10) $\int x^3\sqrt{x^4+8}\,dx$ 10) ______

A) $\frac{2}{3}(x^4+8)^{3/2}+C$

B) $-\frac{1}{2}(x^4+8)^{-1/2}+C$

C) $\frac{1}{6}(x^4+8)^{3/2}+C$

D) $\frac{8}{3}(x^4+8)^{3/2}+C$

put $x^4+8=u$

$du=4x^3dx$

$= \frac{1}{4}\int \sqrt{u}\,du = \frac{1}{4}\cdot\frac{2}{3}u^{3/2}+C$

$= \frac{1}{6}(x^4+8)^{3/2}+C.$

11) $\int \csc^2(9\theta+5)\,d\theta$ 11) ______

A) $18\csc(9\theta+5)\cot(9\theta+5)+C$

B) $9\cot(9\theta+5)+C$

C) $-\frac{1}{9}\cot(9\theta+5)+C$

D) $-\cot(9\theta+5)+C$

put $u=9\theta+5$

$du=9d\theta$

$\int \frac{1}{9}\cdot\csc^2(u)\,d\theta = -\frac{1}{9}\cot u+C$

$= -\frac{1}{9}\cdot\cot(9\theta+5)+C$

Use the substitution formula to evaluate the integral.

12) $\int_{\pi/3}^{2\pi} 3\cos^2 x\sin x\,dx$ 12) ______

put $u=\cos x$

$du=-\sin x\,dx$

A) $\frac{7}{8}$

B) $-\frac{129}{1024}$

C) $-\frac{7}{8}$

D) $-\frac{21}{8}$

$-\int_{\pi/3}^{2\pi} 3u^2du = -u^3\Big/_{\pi/3}^{2\pi} = \cos^3 x\Big/_{2\pi}^{\pi/3} = -\frac{7}{8}$

13) $\int_0^1 \frac{6\,r\,dr}{\sqrt{16+3r^2}}$

13) ______

A) $2\sqrt{19}-8$ B) $-2\sqrt{19}+8$ C) $\frac{\sqrt{19}}{2}-2$ D) $\sqrt{19}-4$

put $u = 16+3r^2$ | at $r=0 \Rightarrow u=16$

$du = 6r\,dr.$ | $r=1 \Rightarrow u=19$

$\int_{16}^{19} \frac{1}{\sqrt{u}}\,du = 2\sqrt{u}\Big|_{16}^{19} = 2\sqrt{19}-8.$

14) $\int_0^{\pi/2} \frac{\cos x}{(4+3\sin x)^3}\,dx$

14) ______

A) $-\frac{33}{1568}$ B) $\frac{33}{1568}$ C) $-\frac{5}{32}$ D) $\frac{11}{1568}$

put $u = 4+3\sin x$

$du = 3\cos x\,dx.$

Do Similarly As Above !!

Find the point(s) at which the given function equals its average value on the given interval.

15) $f(x) = 4 - x^2$; $[-5, 4]$

15) ______

A) $\pm\sqrt{6}$ B) ± 3 C) $\sqrt{5}$ D) $\pm\sqrt{7}$

$\text{Avrg} = \frac{1}{4-(-5)} \int_{-5}^{4} 4-x^2\,dx = \frac{1}{9}\left(4x-\frac{x^3}{3}\right)\Big|_{-5}^{4} = -3$

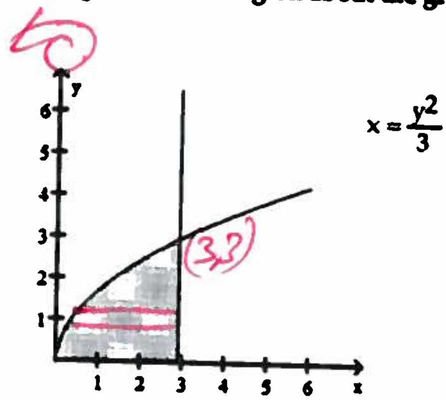
So, $f(x) = -3 \Rightarrow 4-x^2 = -3 \Rightarrow x^2 = 7$

$\Rightarrow x = \pm\sqrt{7}.$

Find the volume of the solid generated by revolving the shaded region about the given axis.

16) About the y-axis

16) ______



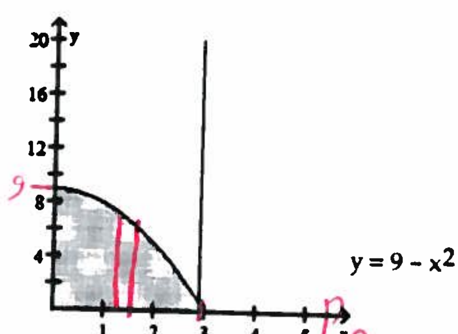
A) $\frac{27}{5}\pi$ B) $\frac{45}{2}\pi$ C) 18π D) $\frac{108}{5}\pi$

$$V=\int_0^3 \pi\left[3^2-\left(\frac{y^2}{3}\right)^2\right]dy = \pi\left[9y-\frac{y^5}{45}\right]_0^3$$

$$= \frac{108}{5}\pi$$

17) About the x-axis

17) ______



A) $\frac{648}{5}\pi$ B) $\frac{1053}{5}\pi$ C) $\frac{3159}{5}\pi$ D) 18π

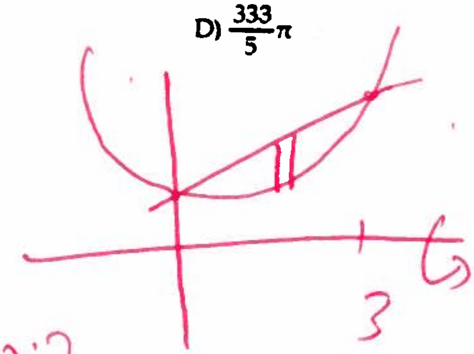
$$V=\pi\int_0^3 (9-x^2)^2\,dx = \pi\int(81-18x^2+x^4)\,dx$$

$$= \pi\left(81x-6x^3+\frac{x^5}{5}\right)\Big|_0^3 = \frac{648}{5}\pi$$

Find the volume of the solid generated by revolving the region bounded by the given lines and curves about the x-axis.

18) $y = x^2 + 1$, $y = 3x + 1$ 18) ______

A) $\frac{63}{2}\pi$ B) $\frac{207}{5}\pi$ C) 27π D) $\frac{333}{5}\pi$



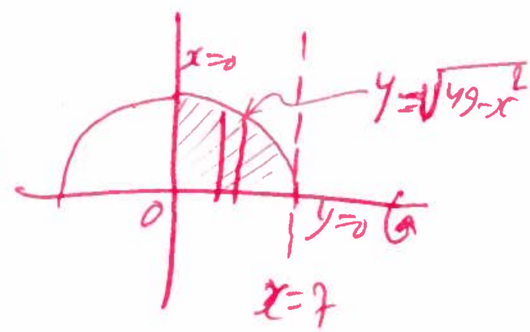
$V = \int_0^3 \pi\left[(3x+1)^2 - (x^2+1)^2\right] dx$

$= \pi \int_0^3 \left[(9x^2+6x+1) - (x^4+2x^2+1)\right] dx$

Do similarly as above, get $\frac{207}{5}\pi$.

19) $y = \sqrt{49 - x^2}$, $y = 0$, $x = 0$, $x = 7$ 19) ______

A) 14π B) $\frac{1372}{3}\pi$ C) $\frac{686}{3}\pi$ D) 196π



$V = \pi \int_0^7 \left(\sqrt{49-x^2}\right)^2 dx = \pi \int_0^7 (49-x^2)\,dx$

$= \pi \left(49x - \frac{x^3}{3}\right)\Big|_0^7 = \frac{686}{3}\pi.$

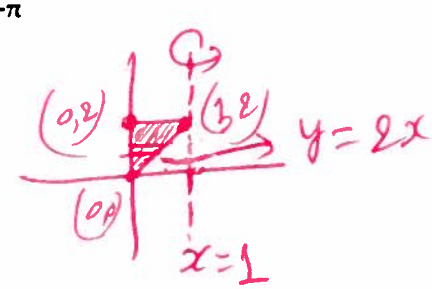
Find the volume of the solid generated by revolving the region about the given axis. Use the shell or washer method.

20) The triangle with vertices (0, 0), (0, 2), and (1, 2) about the line $x = 1$ 20) ______

A) $\frac{5}{3}\pi$ B) $\frac{4}{3}\pi$ C) $\frac{2}{3}\pi$ D) $\frac{1}{3}\pi$

$V = \pi \int_0^2 \left(1 - \left(\frac{y}{2}\right)^2\right) dy = \pi\left[y - \frac{y^3}{12}\right]_0^2$

$= \frac{4\pi}{3}.$



Find the length of the curve.

21) $x = \frac{y^4}{8} + \frac{1}{4y^2}$ from $y = 1$ to $y = 3$ 21) ______

A) $\frac{184}{9}$ B) $\frac{367}{36}$ C) $\frac{41}{4}$ D) $\frac{92}{9}$

$x' = \frac{1}{2}y^3 - \frac{1}{2y}$

$L = \int_1^3 \sqrt{1+(x')^2}\, dy$.

Solve the problem.

22) Given the velocity and initial position of a body moving along a coordinate line at time t, find the body's position at time t. 22) ______

$v = -10t + 5,\ s(0) = 9$

A) $s = 5t^2 + 5t - 9$
B) $s = -10t^2 + 5t + 9$
C) $s = -5t^2 + 5t - 9$
D) $s = -5t^2 + 5t + 9$

$s(t) = s(0) + \int_0^t v(x)dx = 9 + \int_0^t (-10x+5)dx$

$= 9 - \frac{10t^2}{2} + 5t$

$=$

23) Given the acceleration, initial velocity, and initial position of a body moving along a coordinate line at time t, find the body's position at time t. 23) ______

$a = 18 \cos 3t,\ v(0) = 9,\ s(0) = -1$

A) $s = -2 \sin 3t + 9t - 1$
B) $s = -2 \cos 3t + 9t - 1$
C) $s = 2 \cos 3t - 9t - 1$
D) $s = 2 \sin 3t + 9t - 1$

$v(t) = v(0) + \int_0^t 18\cos 3x\,dx = 9 + 6\sin 3t$

So $s(t) = s(0) + \int_0^t (9 + 6\sin 3x)dx$

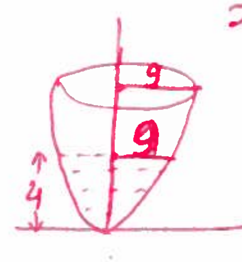
$= -1 + 9t - 2\cos 3t$

24) The hemispherical bowl of radius 9 $=r$ contains water to a depth 4 $=h$. Find the volume of water in the bowl. 24) ______

A) $\frac{368}{3}\pi$ B) $\frac{1826}{3}\pi$ C) $\frac{184}{3}\pi$ D) $\frac{1097}{3}\pi$

$x^2+y^2=81$

$$V = \pi\int_{r-h=5}^{r=9} \left(81-y^2\right)dy.$$



$$= \pi \cdot 81y - \frac{y^3}{3}\Big|_5^9 = \frac{368\pi}{3} \#$$

25) A swimming pool has the shape of a box with a base that measures 20 m by 19 m and a depth of 2 m. How much work is required to pump the water out of the pool when it is full? 25) ______

A) 760,000 J B) 14,896,000 J C) 7,448,000 J D) 28,302,400 J

$$w = \int_0^2 1000\cdot(9.8)\cdot(20\times 19)\,(2-y)\,dy$$

$$= 3724000\left(2y-\frac{y^2}{2}\right)\Big|_0^2$$

$$= 7{,}448{,}000 \text{ J}.$$

Extra (Optional):- What grade are you expecting on this course? ________
Any Feedback .....