# Polynomials and The Exponential Function

**Purpose:** In this project you will investigate the feasibility of approximating the function $e^x$ using polynomials. Although there is a general method using calculus to determine how closely a function can be approximated using polynomials you do not need to know the method to do this project. You will investigate how well polynomials you derive approximate $e^x$.

**Procedure:** You are to follow the steps below. You do not need to do the steps in exactly the order indicated as long as you cover everything you are asked to do. After completing the outline below you should write your results and calculations **neatly**. Be sure to include relevant graphs in what you turn in. Your grade will be determined in part by your presentation.

**Project Outline:** Elmer is writing a computer program for a small business. The program must compute interest compounded continuously, so he needs to use the exponential function. Elmer is using C++ to write the program. In his version of C++, if he includes the standard library of math functions it will increase his code size beyond what would be feasible for the market. He therefore wishes to write his own function in C++ which would give him a good approximation for $e^x$. He wishes to find a polynomial $p(x)$ so that $|p(x) - e^x| < .000005$ for any value of $x$ between $-1$ and 1. Elmer thinks he remembers from his calculus course several years ago that it is possible to find polynomials to approximate $e^x$ to any desired degree of accuracy for **every** real value of $x$. His daughter Anna, who is taking precalculus says that would be impossible. Who is right? Is it possible for Elmer to find a polynomial $p$ with the property given above? If so what is the polynomial?

1. Use the limit $(1 + \frac{x}{n})^n$ and Mathematica to approximate $e^x$ with a polynomial of **very high degree**.
2. Determine what the coefficient of $x^k$ gets close to for each fixed $k$ as $n$ gets very large. Justify your answer.
3. Use the first few limiting coefficients to approxiamte the function $e^x$ with polynomials of fairly low degree. Use Mathematica to find a polynomial of degree as low as you can which will meet the stated condition.
4. Who is right, Elmer or Anna? Either give a polynomial to support Elmer's statement or else give a proof of Anna's statement (assuming the facts covered in class about polynomials).
5. Try to approximate $e$ using what you have done above.
6. Describe how Elmer can use the polynomials you found to approximate $e^x$ accurately for values of $x$ which are not between $-1$ and 1. (Remember that Elmer can only use your polynomial, addition, subtraction, multiplication, and division along with the standard conditionals and loops used in computer programs.)