Lori and Jerry's Trail Optimization Problem

Purpose: This project will give you an introduction to maximizing and minimizing functions. Although you will not learn a general technique, you will use what you know about graphing functions and solving equations to find the minimum of a family of functions.

Procedure: You are to follow the steps below. You do not need to do the steps in exactly the order indicated as long as you cover everything you are asked to do. After completing the outline below you should write your results and calculations **neatly**. Be sure to include relevant graphs in what you turn in. Your grade will be determined in part by your presentation.

Project Outline: Jerry and Lori have been asked to design a trail which goes form a camp site which is at an elevation of 8000 feet up a very steep mountain to an elevation of 10,000 feet. They plan to make the trail spiral up the mountain, which happens to be shaped approximately conical. They are therefore able to give the trail a constant slope all the way to the top. They also decide that they wish to make the trail in such a way that the time it takes for most hikers to get to the top is minimized. After consulting some hiking experts they found that the speed a hiker hikes is proportional to the square of the cosine of the angle of elevation of the path. What angle of elevation should they use to minimize the time it takes to hike the trail?

1. Analyze the problem and find a formula for the hiking time in terms of the angle of elevation.
2. Try to transform your answer in Part 1 so you can solve the problem by finding the maximum or minimum of a polynomial over a range of values for x. (You may wish to think of the problem as **maximizing** one over the time it takes to get to the top.)
3. Use Mathematica to plot graphs to find the value you are seeking in Part 2. Try to guess exactly what the correct answer is. (Hint: square the decimal approximation you get from the graph.)
4. Prove your answer in Part 3 is correct and give Jerry and Lori the answer to their problem. (Hint: To prove your answer you may wish to translate the graph of the polynomial so the point in question moves to the origin. A different approach would be to intersect your graph with a horizontal line through the point in question.)

Note that what you have done in Parts 3 and 4 is to find a turning point of a particular polynomial. The rest of the project is to determine turning points for other polynomials.

5. Find the turning points of the function $x^3 + 4x^2 + 4x - 1$ and prove your answer is correct using the same ideas you used in Parts 2 and 3.
6. Find the formula for the turning points of the function $x^3 + bx^2 + cx + d$. (Hint: Think about what happened in the two examples above. If you are still not sure what to do, try a few more examples of specific polynomials of degree 3 to help you see what to do in general.)
7. **Extra Credit:** Determine an equation that the x-coordinates of all the turning points of a general polynomial $a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$ must satisfy.