Math 1720 Final Review Problems

Note that the final IS COMPREHENSIVE for the entire semester's material but these review problems only cover a sample of problems from sections 8.4, 8.7 and §9. You should also review the earlier sections. For problems on this material I suggest looking again at the earlier review problems, midterms, quizzes, and other problems in the text. In the exam there will be a little more focus on the most recent material, but not too much.

Problems

1. Compute, or show that it does not exist,

$$\int_0^{11} \frac{4}{\sqrt[3]{x-10}}.$$

2. Compute, or show that it does not exist,

$$\int_0^{11} \frac{4}{3x-10}dx$$

2.5. Compute, or show that it does not exist,

$$\int_0^{1} \frac{x}{\sqrt{1-x^4}}dx$$

(Hint: at some point, make a substitution to make the integrand involve a form related to trig subs.) Use symmetry to deduce what $\int_{-1}^{1}$ is, with the same integrand.

3.(a) Find

$$\int_4^5 \frac{x^3-x-3}{(x-3)x^3}dx$$

(b) Find

$$\int_2^3 \frac{x}{x^3-1}dx$$

4. Let $\{a_n\}_{n=5}^{\infty}$ be the sequence with terms

$$a_n = \int_0^{2n} 4^{-x+10}dx.$$

Determine whether $\lim_{n\to\infty} a_n$ exists, and if so, its value. How can you write this as an improper integral?

5. Find

$$\int_{-\infty}^{\infty} \frac{e^x}{e^{2x}+1}dx$$

6. Find

$$\int_{10}^{\infty} \frac{1}{x\ln(x^3)}dx$$

7.(a) Given the recurrence $a_1 = 3$, $a_{n+1} = \frac{a_n - 1}{a_n}$, find the first eight terms of the sequence. Does $\lim_{n\to\infty} a_n$ exist? Explain.

(b) Find a recurrence, and a general formula for a_n, for the sequence

$$4, 7, 12, 19, 28, 39, \ldots,$$

assuming that the first index is $n = 1$. (Hint: for the general formula, try subtracting 3 from every term.)

(c) Find a formula for a_n, for the sequence

$$4, 5 + \frac{1}{2}, 4 + \frac{3}{4}, 5 + \frac{1}{8}, 4 + \frac{15}{16}, \ldots$$

Based on your formula, does the limit of the sequence exist? Explain.

8.(a) If the sequence $a_0, a_1, a_2, \ldots$ is geometric and $a_1 = 5$ and $a_3 = 15$, what can you say about the ratio r of the sequence? What is the value of a_{101}?

(b) If the sequence $a_0, a_1, a_2, \ldots$ is such that $a_1 = 5$ and $a_3 = 15$, what can you say about a_{101}?

9. Suppose $\{c_n\}_{n=3}^{\infty}$ is a sequence such that $\lim_{n\to\infty} c_n = 6$, and $\{d_n\}_{n=3}^{\infty}$ has limit 3. In problems (a*), (a) and (b), compute the limit of the sequence, if possible, where in (a*) the sequence has the terms e_n shown, in (a), the terms a_n and in (b), the terms b_n. Do part (c).

(a*) The sequence with terms

$$e_n = n d_n^3 - \frac{n^2 d_n^3 - d_n}{n+1}, \quad n \geq 3$$

(a) The sequence with terms

$$a_n = n d_n - \frac{(n^2 - 5n + 1)c_n}{2n}, \quad n \geq 3$$

(b) The sequence with terms

$$b_n = n^{\frac{1}{n^3}}, \quad n \geq 1$$

(c) Determine whether

$$\sum_{n=3}^{\infty} d_n / c_n$$

converges.

10.(a) Find the first four partial sums for the series

$$\sum_{n=0}^{\infty} \frac{1}{7 \cdot 3^n}$$

(b) Evaluate the sum

$$\sum_{n=0}^{1{,}000{,}000} \frac{1}{7 \cdot 3^n},$$

simplifying fully.

(c) Does the series in (a) converge? If so, find the value it converges to.

(d) Repeat (c) for

$$\sum_{n=0}^{\infty}(-7/4)^n(3/4)^n$$

(e) Repeat (c) for

$$\sum_{n=0}^{\infty}500(-7/4)^n(3/4)^{2n}$$