Math 1720 Homework 3, due Friday Feb 10
Explain all answers and show all calculations.

Note: for the starred problems, see the modifications in the notes below. The lettered problems A, B, C are given below.

7.1: 15*, 18*, A, 21, 26, 27, 28*, 30, 31, 33, 36, 39, B; (C is optional.)

Notes:
15*: also graph f and f^{-1} on the same set of axes, and make sure the graphs have the correct symmetry with one another.
18*: Also find the inverse over the interval $x \leq 0$. For each of the intervals ($x \geq 0$ and $x \leq 0$) also find the domain and range of the inverse for that interval.
28*: Also do this problem with the interval $x < 3$. For each of the intervals ($x < 3$ and $x > 3$) also find the domain and range of the inverse for that interval.

Problems A, B, C:
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A. Find the longest intervals over which

$$f(x) = x^2 + 2x - 8$$

is 1-1 (one-to-one). Find the range of f over each of these intervals. Find the inverse of f over each of these intervals, and also find the inverse's domain and range.

B. Suppose f is differentiable and 1-1 and has domain $(0, \infty)$ and range $(-\infty, 10)$. Let f^{-1} be the inverse of f over D. Suppose that all the following equations are true:

- $f(3) = 6$ and $f'(3) = -0.5$
- $f(4) = 5$ and $f'(4) = -1.5$
- $f(5) = 3$ and $f'(5) = -2$
- $f(6) = 2$ and $f'(6) = 0$

(a) How do we know f^{-1} exists? Find the domain and range of f^{-1}.
(b) Is f increasing over $(0, \infty)$? Is f decreasing over $(0, \infty)$? Or is there insufficient information to determine this?

(c) Do you have enough information to find the equation for the tangent to the graph of $y = f^{-1}(x)$ at:

(i) $x = 4$?

(ii) $x = 3$?

If so, find the equation; if not, explain why not.

(d) Is f^{-1} differentiable at $x = 2$? (hint: work out the behaviour of the graph there.)

C (not required). Consider the points $P = (a, b)$ and $Q = (b, a)$ in the plane. Let R be the point of intersection between the line PQ and the line $y = x$. Show that $R = (\frac{a+b}{2}, \frac{a+b}{2})$, and that the distance from P to R equals the distance from Q to R. (So combined with what we showed in class Friday, that PQ is perpendicular to $y = x$, we get that P and Q are mirror images of one another in the "mirror" line $y = x$.)