Intro to topology homework 12. Due Friday April 27.
Please give to Steve Jackson.

1. *Do parts (a), (b), (c) of this problem without using path-connectedness.* (Use various theorems presented in class on (standard) connectedness.)

(a) Let $p, q, r \in \mathbb{R}^3$ be three distinct points. Show that $\mathbb{R}^3 \setminus \{p, q, r\}$ is connected. (In fact, a similar proof works for any finite collection of points in $\mathbb{R}^3$, which you may now assume.)

(b) S_2 denotes the surface of the sphere. Prove that S_2 is connected.

(c) Let $p \in S_2$. Show that $S_2 \setminus \{p\}$ is connected.

(d)* (Extra credit.) Let A be the union of three lines in $\mathbb{R}^3$. Prove that $\mathbb{R}^3 \setminus A$ is path-connected.

2. Use the characterization of closure given in class and in the midterm 2 review problems, for this problem.

(a) Let $X = \mathbb{N}$ and τ be the cofinite topology. Let A be the set of even elements of $\mathbb{N}$. Compute the closure of A.

(b) (With correction made) Let (X, τ) be a top space and $A \subseteq X$. For $B \subseteq A$, let $\mathrm{Cl}_X(B)$ be the closure of B w.r.t. τ and let $\mathrm{Cl}_A(B)$ be the closure of B w.r.t. $\tau \restriction A$. Prove that for $B \subseteq A$, $\mathrm{Cl}_A(B) = \mathrm{Cl}_X(B) \cap A$. Prove that if A is closed iff for all $B \subseteq A$ we have $\mathrm{Cl}_X(B) = \mathrm{Cl}_A(B)$.

(c) Let $X = \mathbb{R}^2$ and let τ be the topology on $\mathbb{R}^2$ with the following base $\mathfrak{b}$: $\mathfrak{b} = \tau_{\mathrm{std},2} \cup \sigma$, where (i) $\tau_{\mathrm{std},2}$ is the standard topology on $\mathbb{R}^2$, and (ii) σ is the subspace topology on the x-axis.

Now let $A \subseteq \mathbb{R}^2$. Let $Z \subseteq \mathbb{R}^2$ be the x-axis. Let Cl be closure w.r.t. τ, Cl_2 closure w.r.t. the standard topology on $\mathbb{R}^2$. Show that

$$\mathrm{Cl}(A) = (\mathrm{Cl}_2(A) \setminus Z) \cup \mathrm{Cl}_2(A \cap Z).$$

(Try stating this fact more generally.)

These exercises are from Munkres:
§23: 1, 2, 6(note), 9(note)

In §23: 6, $Bd(A)$ is $\mathrm{Cl}(A) \setminus \mathrm{Int}(A)$.
§23: 9 is not required for 4500 (but can be completed for extra credit)