Intro to Topology Homework 5 - due Wednesday.
(Note: all solutions, including examples, should be explained, unless stated otherwise.)
(I'll discuss continuity a little more on Tuesday.)

1. Let (X, d) be a metric space and fix $a \in X$. Define $F : X \to \mathbb{R}$ by $F(x) = 10^3 d(x, a)$. Show that F is continuous.

2. Let $X = C([0, 1])$ and let $F : X \to \mathbb{R}$ be defined by $F(f) = \int_0^1 f(x)dx$. Show that F is continuous as a function from $(X, d_{\max})$ to $\mathbb{R}$. (It is also continuous as a function from $(X, d_{\int})$ to $\mathbb{R}$.)

3. Let (X, d) and (Y, e) be metric spaces and $f : X \to Y$. We say f is *Lipschitz continuous* iff there is $K \geq 0$ such that for all $x, y \in X$,

$$e(f(x), f(y)) \leq Kd(x, y).$$

Prove that if f is Lipschitz continuous then f is continuous.

Give an example of a continuous function $f : \mathbb{R} \to \mathbb{R}$ which is not Lipschitz continuous. (Hint: Think about what Lipschitz continuity means with regard to f', if f is differentiable.)