Intro to Topology Homework 8 - due Friday Mar 16.
(Note: all solutions, including examples, should be explained, unless stated otherwise.)

Let C be the Cantor set.

1. Prove that the two definitions of the Cantor set given in class are equivalent.

2. Show that there is a non-decreasing, continuous function $f : [0,1] \to [0,1]$ such that $f``C = [0,1]$ (in particular, f is surjective) and for all intervals I, if $I \subseteq [0,1]\backslash C$, then f is constant on I. Show that $f \restriction C$ is not 1-1.

3. (a) We saw in class that C is uncountable. Explain how the proof can be modified to show that for every open interval I, if $C \cap I \neq \emptyset$, then $C \cap I$ is uncountable. (You don't need to give the full proof, just the modifications.)

(b) Suppose that $A \subseteq \mathbb{R}$, A is closed, $A \neq \emptyset$, and no point in A is isolated (i.e. for every $x \in A$, for every $\varepsilon > 0$, $(x-\varepsilon, x+\varepsilon) \cap A$ includes a point $\neq x$). Show that A is uncountable. (Hint: give a proof similar to that for $\mathbb{R}$ and the Cantor set.)

4.

(a) Let X, Y be two uncountable disjoint sets. Let τ be the collection of sets $A \subseteq (X \cup Y)$ such that $A \cap X$ is countable or $X \backslash A$ is countable. Show that (X, τ) is a topological space.

(b) Let X be a 3-element set. Describe all topologies on X. Explain which topologies are coarser/finer than others. (Some of the topologies are isomorphic to others. You should group such topologies together, and explain them once, but specify how many ways they can be achieved on this particular set.)