We can now express the areas of the top, bottom, and sides in terms of r only.

In Words	In Algebra
Radius of can	r
Height of can	$\dfrac{1000}{\pi r^2}$
Area of top and bottom	$2\pi r^2$
Area of sides ($2\pi rh$)	$2\pi r\left(\dfrac{1000}{\pi r^2}\right)$

■ **Set up the Model**

The model is the function S that gives the surface area of the can as a function of the radius r.

$$\text{surface area} = \text{area of top and bottom} + \text{area of sides}$$

$$S(r) = 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$$

$$S(r) = 2\pi r^2 + \frac{2000}{r}$$

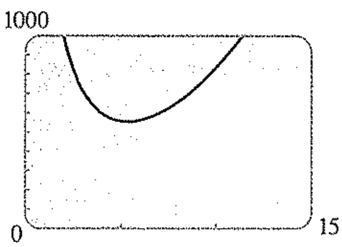


Figure 9

$S = 2\pi r^2 + \dfrac{2000}{r}$

■ **Use the Model**

We use the model to find the minimum surface area of the can. We graph S in Figure 9 and zoom in on the minimum point to find that the minimum value of S is about 554 cm^2 and occurs when the radius is about 5.4 cm. ■

2.6 Exercises

1–18 ■ In these exercises you are asked to find a function that models a real-life situation. Use the guidelines for modeling described in the text to help you.

1. **Area** A rectangular building lot is three times as long as it is wide. Find a function that models its area A in terms of its width w.

2. **Area** A poster is 10 inches longer than it is wide. Find a function that models its area A in terms of its width w.

3. **Volume** A rectangular box has a square base. Its height is half the width of the base. Find a function that models its volume V in terms of its width w.

4. **Volume** The height of a cylinder is four times its radius. Find a function that models the volume V of the cylinder in terms of its radius r.

5. **Area** A rectangle has a perimeter of 20 ft. Find a function that models its area A in terms of the length x of one of its sides.

6. **Perimeter** A rectangle has an area of 16 m^2. Find a function that models its perimeter P in terms of the length x of one of its sides.

7. **Area** Find a function that models the area A of an equilateral triangle in terms of the length x of one of its sides.

8. **Area** Find a function that models the surface area S of a cube in terms of its volume V.

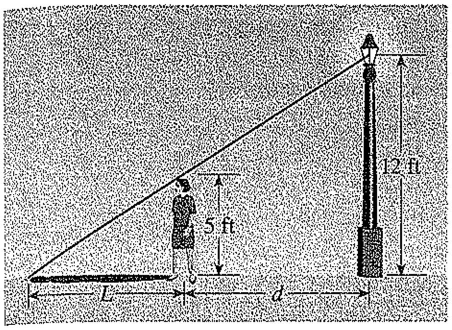
9. **Radius** Find a function that models the radius r of a circle in terms of its area A.

10. **Area** Find a function that models the area A of a circle in terms of its circumference C.

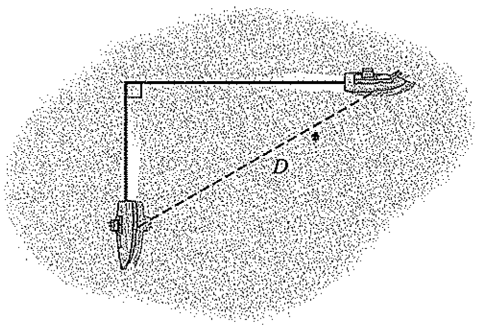
11. **Area** A rectangular box with a volume of 60 ft^3 has a square base. Find a function that models its surface area S in terms of the length x of one side of its base.

12. **Length** A woman 5 ft tall is standing near a street lamp that is 12 ft tall, as shown in the figure. Find a function that

models the length L of her shadow in terms of her distance d from the base of the lamp.



13. **Distance** Two ships leave port at the same time. One sails south at 15 mi/h and the other sails east at 20 mi/h. Find a function that models the distance D between the ships in terms of the time t (in hours) elapsed since their departure.

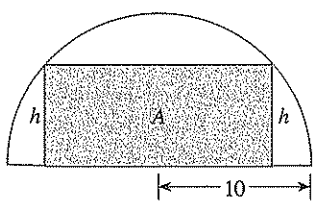


14. **Product** The sum of two positive numbers is 60. Find a function that models their product P in terms of x, one of the numbers.

15. **Area** An isosceles triangle has a perimeter of 8 cm. Find a function that models its area A in terms of the length of its base b.

16. **Perimeter** A right triangle has one leg twice as long as the other. Find a function that models its perimeter P in terms of the length x of the shorter leg.

17. **Area** A rectangle is inscribed in a semicircle of radius 10, as shown in the figure. Find a function that models the area A of the rectangle in terms of its height h.



18. **Height** The volume of a cone is 100 in^3. Find a function that models the height h of the cone in terms of its radius r.

19–36 ▪ In these problems you are asked to find a function that models a real-life situation, and then use the model to answer questions about the situation. Use the guidelines on page 205 to help you.

19. **Maximizing a Product** Consider the following problem: Find two numbers whose sum is 19 and whose product is as large as possible.
    (a) Experiment with the problem by making a table like the one below, showing the product of different pairs of numbers that add up to 19. Based on the evidence in your table, estimate the answer to the problem.

First number	Second number	Product
1	18	18
2	17	34
3	16	48
⋮	⋮	⋮

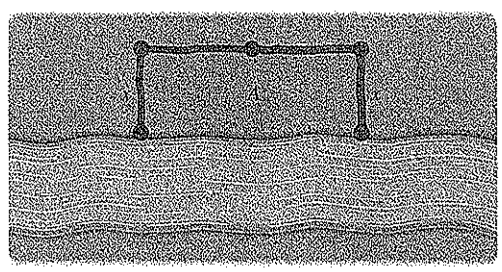
    (b) Find a function that models the product in terms of one of the two numbers.
    (c) Use your model to solve the problem, and compare with your answer to part (a).

20. **Minimizing a Sum** Find two positive numbers whose sum is 100 and the sum of whose squares is a minimum.

21. **Maximizing a Product** Find two numbers whose sum is -24 and whose product is a maximum.

22. **Maximizing Area** Among all rectangles that have a perimeter of 20 ft, find the dimensions of the one with the largest area.

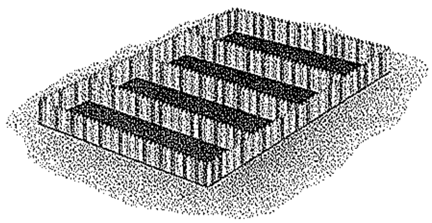
23. **Fencing a Field** Consider the following problem: A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He does not need a fence along the river (see the figure). What are the dimensions of the field of largest area that he can fence?
    (a) Experiment with the problem by drawing several diagrams illustrating the situation. Calculate the area of each configuration, and use your results to estimate the dimensions of the largest possible field.
    (b) Find a function that models the area of the field in terms of one of its sides.
    (c) Use your model to solve the problem, and compare with your answer to part (a).



24. Dividing a Pen A rancher with 750 ft of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle (see the figure).

(a) Find a function that models the total area of the four pens.

(b) Find the largest possible total area of the four pens.

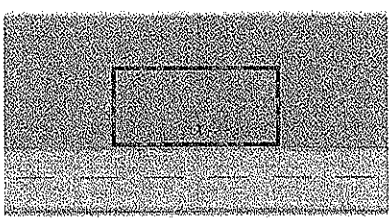


25. Fencing a Garden Plot A property owner wants to fence a garden plot adjacent to a road, as shown in the figure. The fencing next to the road must be sturdier and costs \$5 per foot, but the other fencing costs just \$3 per foot. The garden is to have an area of 1200 ft^2.

(a) Find a function that models the cost of fencing the garden.

(b) Find the garden dimensions that minimize the cost of fencing.

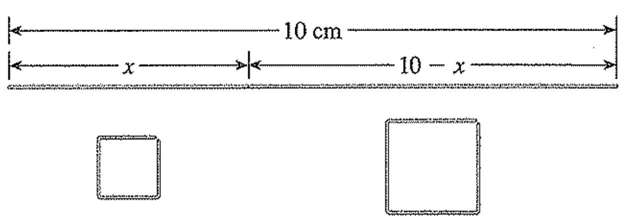
(c) If the owner has at most \$600 to spend on fencing, find the range of lengths he can fence along the road.



26. Maximizing Area A wire 10 cm long is cut into two pieces, one of length x and the other of length $10 - x$, as shown in the figure. Each piece is bent into the shape of a square.

(a) Find a function that models the total area enclosed by the two squares.

(b) Find the value of x that minimizes the total area of the two squares.



27. Stadium Revenue A baseball team plays in a stadium that holds 55,000 spectators. With the ticket price at \$10, the average attendance at recent games has been 27,000. A market survey indicates that for every dollar the ticket price is lowered, attendance increases by 3000.

(a) Find a function that models the revenue in terms of ticket price.

(b) What ticket price is so high that no revenue is generated?

(c) Find the price that maximizes revenue from ticket sales.

28. Maximizing Profit A community bird-watching society makes and sells simple bird feeders to raise money for its conservation activities. The materials for each feeder cost \$6, and they sell an average of 20 per week at a price of \$10 each. They have been considering raising the price, so they conduct a survey and find that for every dollar increase they lose 2 sales per week.

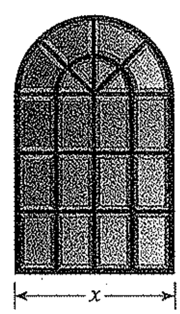
(a) Find a function that models weekly profit in terms of price per feeder.

(b) What price should the society charge for each feeder to maximize profits? What is the maximum profit?

29. Light from a Window A Norman window has the shape of a rectangle surmounted by a semicircle, as shown in the figure. A Norman window with perimeter 30 ft is to be constructed.

(a) Find a function that models the area of the window.

(b) Find the dimensions of the window that admits the greatest amount of light.

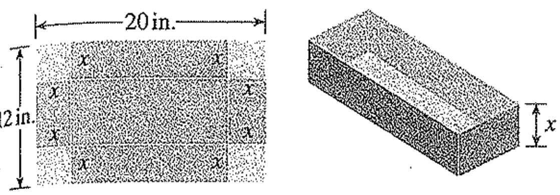


30. Volume of a Box A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 in. by 20 in. by cutting out equal squares of side x at each corner and then folding up the sides (see the figure).

(a) Find a function that models the volume of the box.

(b) Find the values of x for which the volume is greater than 200 in^3.

(c) Find the largest volume that such a box can have.

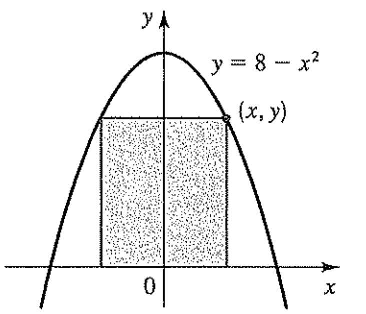


31. Area of a Box An open box with a square base is to have a volume of 12 ft^3.

(a) Find a function that models the surface area of the box.

(b) Find the box dimensions that minimize the amount of material used.

32. Inscribed Rectangle Find the dimensions that give the largest area for the rectangle shown in the figure. Its base is on the x-axis and its other two vertices are above the x-axis, lying on the parabola $y = 8 - x^2$.



33. Minimizing Costs A rancher wants to build a rectangular pen with an area of 100 m^2.

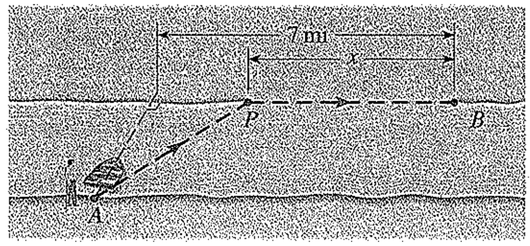
(a) Find a function that models the length of fencing required.

(b) Find the pen dimensions that require the minimum amount of fencing.

34. Minimizing Time A man stands at a point A on the bank of a straight river, 2 mi wide. To reach point B, 7 mi downstream on the opposite bank, he first rows his boat to point P on the opposite bank and then walks the remaining distance x to B, as shown in the figure. He can row at a speed of 2 mi/h and walk at a speed of 5 mi/h.

(a) Find a function that models the time needed for the trip.

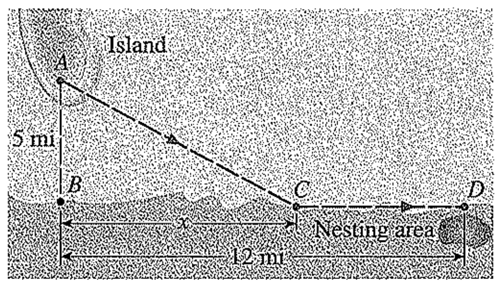
(b) Where should he land so that he reaches B as soon as possible?



35. Bird Flight A bird is released from point A on an island, 5 mi from the nearest point B on a straight shoreline. The bird flies to a point C on the shoreline, and then flies along the shoreline to its nesting area D (see the figure). Suppose the bird requires 10 kcal/mi of energy to fly over land and 14 kcal/mi to fly over water (see Example 9 in Section 1.6).

(a) Find a function that models the energy expenditure of the bird.

(b) If the bird instinctively chooses a path that minimizes its energy expenditure, to what point does it fly?



36. Area of a Kite A kite frame is to be made from six pieces of wood. The four pieces that form its border have been cut to the lengths indicated in the figure. Let x be as shown in the figure.

(a) Show that the area of the kite is given by the function

$$A(x) = x(\sqrt{25 - x^2} + \sqrt{144 - x^2})$$

(b) How long should each of the two crosspieces be to maximize the area of the kite?