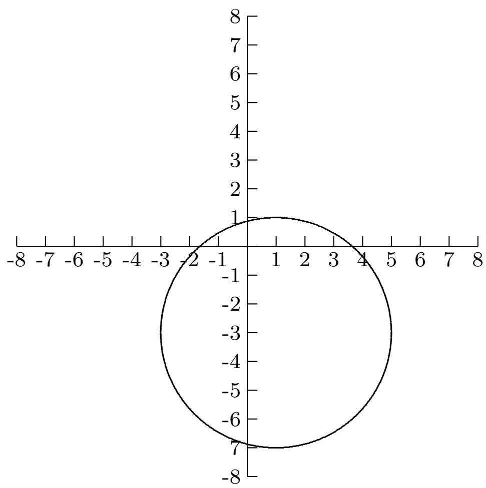
MATH 1100 FINAL EXAM

Fall 2009

Section I–Multiple Choice

1. Find an equation of the circle whose graph is below.



   (a) $(x+1)^2 + (y-3)^2 = 4$
   (b) $(x-1)^2 + (y+3)^2 = 4$
   (c) $(x-1)^2 + (y+3)^2 = 16$
   (d) $(x+1)^2 + (y-3)^2 = 16$

2. If $(a, -7)$ is a point on the graph of $y = 2x - 5$, what is a?

   (a) 4
   (b) -1
   (c) 1
   (d) -4

3. Find all points having an y-coordinate of 4 whose distance from the point $(3, -2)$ is 10.

   (a) $(13, 4)$, $(6, 4)$
   (b) $(13, 4)$, $(-7, 4)$
   (c) $(-5, 4)$, $(11, 4)$
   (d) $(6, 4)$, $(-5, 4)$

4. Find an equation for the line with the given properties.
   Perpendicular to the line $4x + y = 7$, containing the point $(0, \frac{7}{4})$.

(a) $y = \frac{1}{4}x + 7$
(b) $y = -\frac{1}{4}x + \frac{7}{4}$
(c) $y = \frac{3}{2}$
(d) $y = \frac{1}{4}x + \frac{7}{4}$

5. Let $f(x) = 2x^2 + 3$. Compute $\frac{f(a+h)-f(a)}{h}$.

(a) 1

(b) $4a + 2h$

(c) $4ah + 2$

(d) $4a + h^2$

6. Consider the function $f(x) = 3x^4 + 9x^2 + 7$. The Rational Zero Theorem eliminates which of the following as a possible zero?

(a) -1
(b) $\frac{3}{7}$
(c) $-\frac{1}{3}$
(d) 7

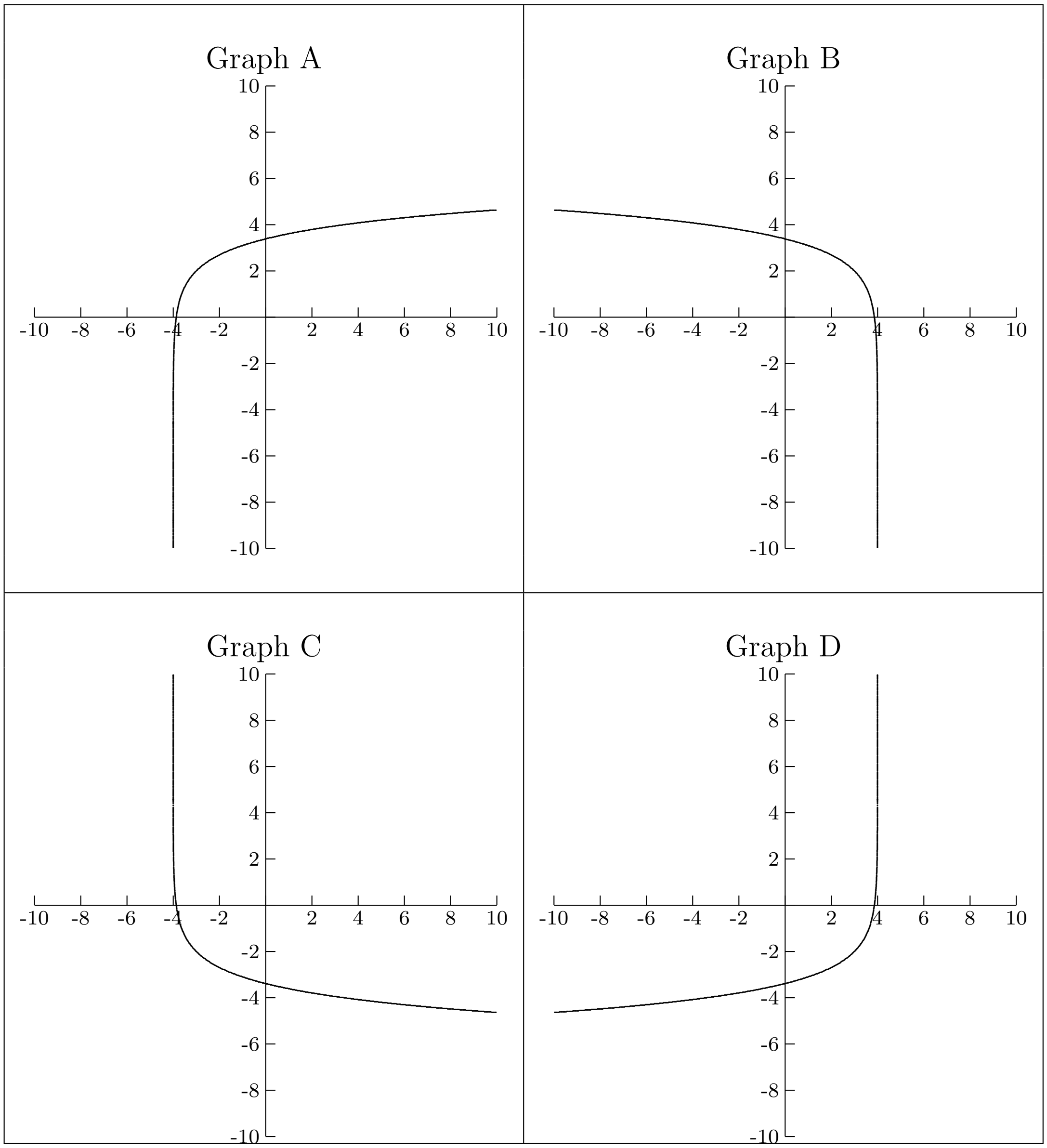
7. The marketing manager at a jeans company wishes to find a function that relates the demand D for men's jeans and p, the price of the jeans. The following data were obtained based on a price history of the jeans. Express the relationship as a linear function (use the linear regression function on your calculator.)

P	p
19	60
21	57
22	56
22	53
26	52
28	49
29	44

(a) $P(d) = -1.336d + 84.862$
(b) $D(p) = -1.336p + 84.862p$
(c) $P(d) = -1.336f(x) + 84.862$

(d) $D(p) = -1336p + 84.862$

8. Use transformations of the graph of $y = \ln x$ to sketch the graph of $f(x) = 2 + \ln(4 - x)$.



9. Find k such that $f(x) = x^4 + kx^3 + 2$ has the factor $x - 1$.

(a) 2

(b) -2

(c) 3

(d) -3

10. Find the horizontal and vertical asymptotes of the following rational function.

$$r(x) = \frac{7x+1}{x-8}$$

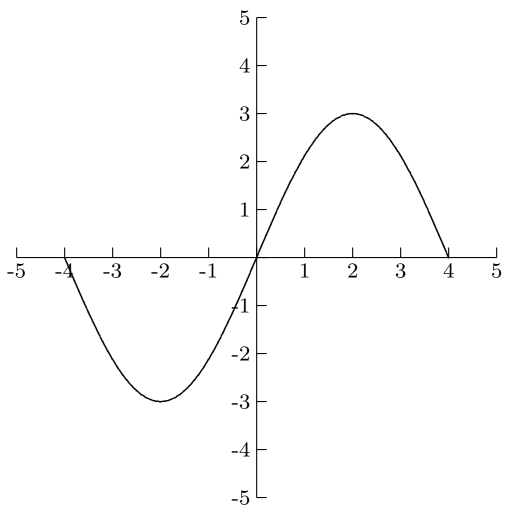
(a) horizontal asymptote $y = 7$; vertical asymptote $x = 8$

(b) horizontal asymptote $y = 7$; vertical asymptote $x = -8$

(c) horizontal asymptote $y = 0$; vertical asymptote $x = -16$

(d) horizontal asymptote $y = 0$; vertical asymptote $x = 8$

(e) horizontal asymptote $y = 0$; vertical asymptote $x = -8$



11. What is the domain of this function?

(a) $[-4, 3]$

(b) $[-4, 4]$

(c) $[-3, 3]$

(d) $[-3, 4]$

12. For the given functions f and g, find the requested composite function value.

$$f(x) = \frac{x-6}{x}, \; g(x) = x^2 + 9, \quad \text{Find } (f \circ g)(-2).$$

(a) $\frac{7}{13}$

(b) 25

(c) $\frac{145}{16}$

(d) 13

13. The function $A = A_0 e^{-0.00693x}$ models the weight (in pounds) of a particular radioactive material stored in a concrete vault, where x is the number of years since A_0 pounds of the material was put into the vault. If 500 pounds of the material is initially put into the vault, how many pounds will be left after 175 years?

    (a) 100 lbs
    (b) 425 lbs
    (c) 154 lbs
    (d) 149 lbs

14. Solve the equation

$$\frac{3}{x+5} + \frac{2}{2x+1} = \frac{4}{x-2}$$

    (a) $\frac{47}{46}$
    (b) $-\frac{46}{47}$
    (c) $\frac{46}{47}$
    (d) $\frac{-47}{46}$

15. A motorboat maintained a constant speed of 18 miles per hour relative to the water in going 40 miles upstream and then returning. The total time for the trip was 4.5 hours. Use this information to find the speed of the current in miles per hour.

    (a) 3 miles per hour
    (b) 5 miles per hour
    (c) 7 miles per hour
    (d) 4 miles per hour
    (e) 2 miles per hour

16. Solve the system of equations.

$$\begin{cases} 2x + 4y - 2z = 0 \\ 3x + 5y = 1 \end{cases}$$

    (a) $x = 2 + 7z$, and $y = -3z - 1$, where z is any real number
    (b) $x = -2 - 7z$, and $y = -3z - 1$, where z is any real number
    (c) $x = 2 - 5z$, and $y = 3z - 1$, where z is any real number
    (d) $x = 1 - 5z$, and $y = 3z - \frac{1}{2}$, where z is any real number

Section 2–Free Response

17. Find $f^{-1}(x)$ (the inverse function) if $f(x) = \frac{1}{3x-2}$.

18. Solve the equation.

$$x + \sqrt{x+44} = 12$$

19. Solve the equation by completing the square. If you solve the equation using any other method, you will receive **no** credit.

$$x^2 - 4x - 8 = 0$$

20. Suppose that the quantity supplied S and quantity demanded D of T-shirts at a concert are given by the following functions, where p is the price per T-shirt.

$$S(p) = -250 + 60p$$
$$D(p) = 1370 - 75p$$

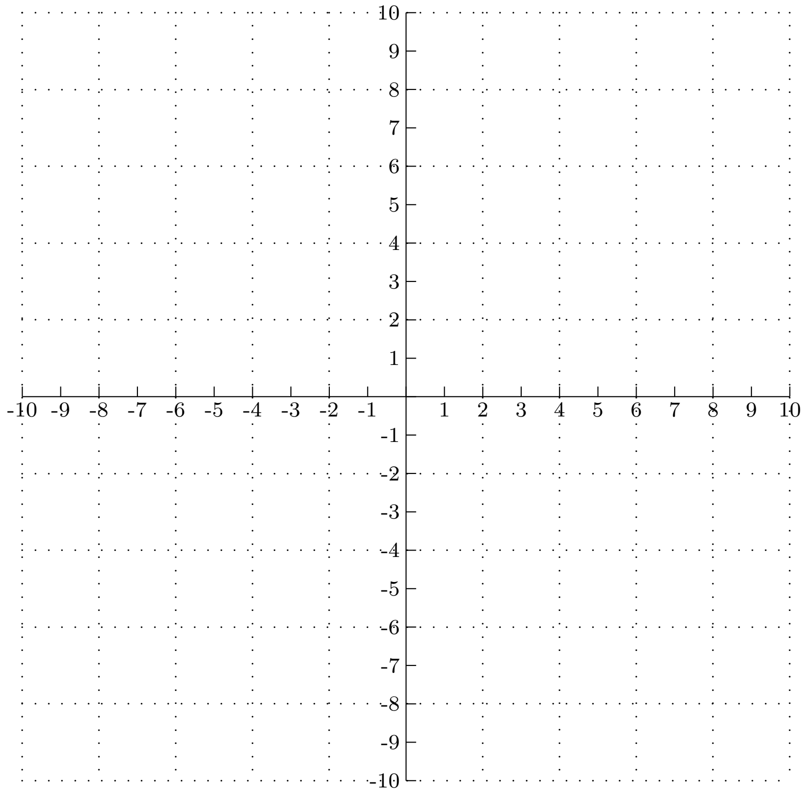
Find the equilibrium price p (defined as the price at which quantity supplied equals quantity demanded).

21. Solve the equation.

$$\log_2(7x+4) - \log(x-2) = 4$$

22. Graph the solution to the system of inequalities. (There is no work that you need to show.)

$$\begin{cases} x - y < 4 \\ y < -x^2 + 2x^+8 \end{cases}$$



10
9
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-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 1 2 3 4 5 6 7 8 9 10
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10

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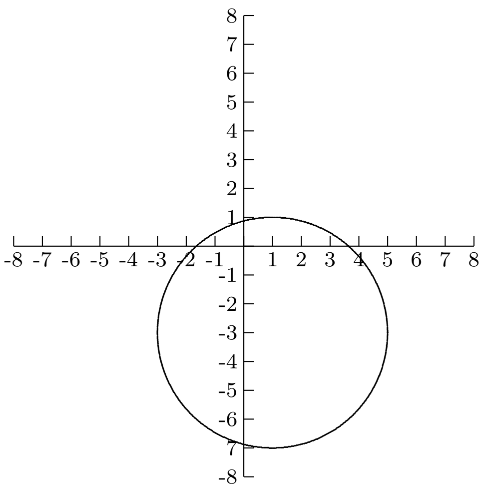
Section I–Multiple Choice

1. Find an equation of the circle whose graph is below.



Solution

The center of the circle is $(1, -3)$, so $h = 1$ and $k = -3$. Putting these into the standard equation of a circle gives us $(x-1)^2 + (y-(-3))^2 = r^2$. The radius is 4, so we now have the equation of the circle: $(x-1)^2 + (y+3)^2 = 16$.

(a) $(x+1)^2 + (y-3)^2 = 4$
(b) $(x-1)^2 + (y+3)^2 = 4$
(c) $(x-1)^2 + (y+3)^2 = 16$
(d) $(x+1)^2 + (y-3)^2 = 16$

2. If $(a, -7)$ is a point on the graph of $y = 2x - 5$, what is a?

(a) 4
(b) -1
(c) 1
(d) -4

Solution

Substitute a for x and -7 for y and solve for a.

$$-7 = 2a - 5$$
$$-2 = 2a$$
$$-1 = a$$

3. Find all points having an y-coordinate of 4 whose distance from the point $(3, -2)$ is 10.

   (a) $(13, 4)$, $(6, 4)$
   (b) $(13, 4)$, $(-7, 4)$
   (c) $(-5, 4)$, $(11, 4)$
   (d) $(6, 4)$, $(-5, 4)$

   Solution
   In $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, $d = 10$, $x_1 = 3$, $y_1 = -2$,and $y_2 = 4$. We now find x_1.

$$
\begin{aligned}
10 &= \sqrt{(x_2 - 3)^2 + (4 - (-2))^2} \\
100 &= (x_2 - 3)^2 + (4 - (-2))^2 && \text{Square each side.} \\
64 &= (x_2 - 3)^2 \\
\pm 8 &= x_2 - 3 && \text{Take the square root of each side.} \\
x_2 &= 3 - 8,\ 3 + 8 = 11, -5
\end{aligned}
$$

   The points are $(11, 4)$ and $(-5, 4)$.

4. Find an equation for the line with the given properties.
   Perpendicular to the line $4x + y = 7$, containing the point $(0, \frac{7}{4})$.

   (a) $y = \frac{1}{4}x + 7$
   (b) $y = -\frac{1}{4}x + \frac{7}{4}$
   (c) $y = \frac{3}{2}$
   (d) $y = \frac{1}{4}x + \frac{7}{4}$

   Solution
   We find the slope of $4x + y = 7$ and compute the negative reciprocal of this number. The given point is the y-intercept (because $x = 0$), so we then would have both m and b in $y = mx + b$.

$$
\begin{aligned}
4x + y &= 7 \\
y &= -4x + 7
\end{aligned}
$$

   The slope of this line is -4. The slope of the line we want is $\frac{1}{4}$. This gives us $y = \frac{1}{4}x + b$. Because $b = \frac{7}{4}$, we have the equation of the line $y = \frac{1}{4}x + \frac{7}{4}$.

5. Let $f(x) = 2x^2 + 3$. Compute $\frac{f(a+h)-f(a)}{h}$.

(a) 1

(b) $4a + 2h$

(c) $4ah + 2$

(d) $4a + h^2$

Solution

$$\begin{aligned}
f(a+h) &= 2(a+h)^2 + 3 = 2(a^2 + 2ah + h^2) + 3 = 2a^2 + 4ah + 2h^2 + 3 \\
f(a) &= 2a^2 + 3 \\
\frac{f(a+h) - f(a)}{h} &= \frac{2a^2 + 4ah + 2h^2 + 3 - (2a^2 + 3)}{h} \\
&= \frac{2a^2 + 4ah + 2h^2 + 3 - 2a^2 - 3}{h} \\
&= \frac{4ah + 2h^2}{h} \\
&= \frac{h(4a + 2h)}{h} \\
&= 4a + 2h
\end{aligned}$$

6. Consider the function $f(x) = 3x^4 + 9x^2 + 7$. The Rational Zero Theorem eliminates which of the following as a possible zero?

(a) -1

(b) $\frac{3}{7}$

(c) $-\frac{1}{3}$

(d) 7

Solution

According to the Rational Zero Theorem, the numerator of a possible rational zero divides the constant term, in this case 7, and the denominator divides the leading coefficient, in this case, 3. This means that $\frac{3}{7}$ is not a possible rational zero.

7. The marketing manager at a jeans company wishes to find a function that relates the demand D for men's jeans and p, the price of the jeans. The following data were obtained based on a price history of the jeans. Express the relationship as a linear function (use the linear regression function on your calculator.)

P	p
19	60
21	57
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(a) $P(d) = -1.336d + 84.862$

(b) $D(p) = -1.336p + 84.862p$

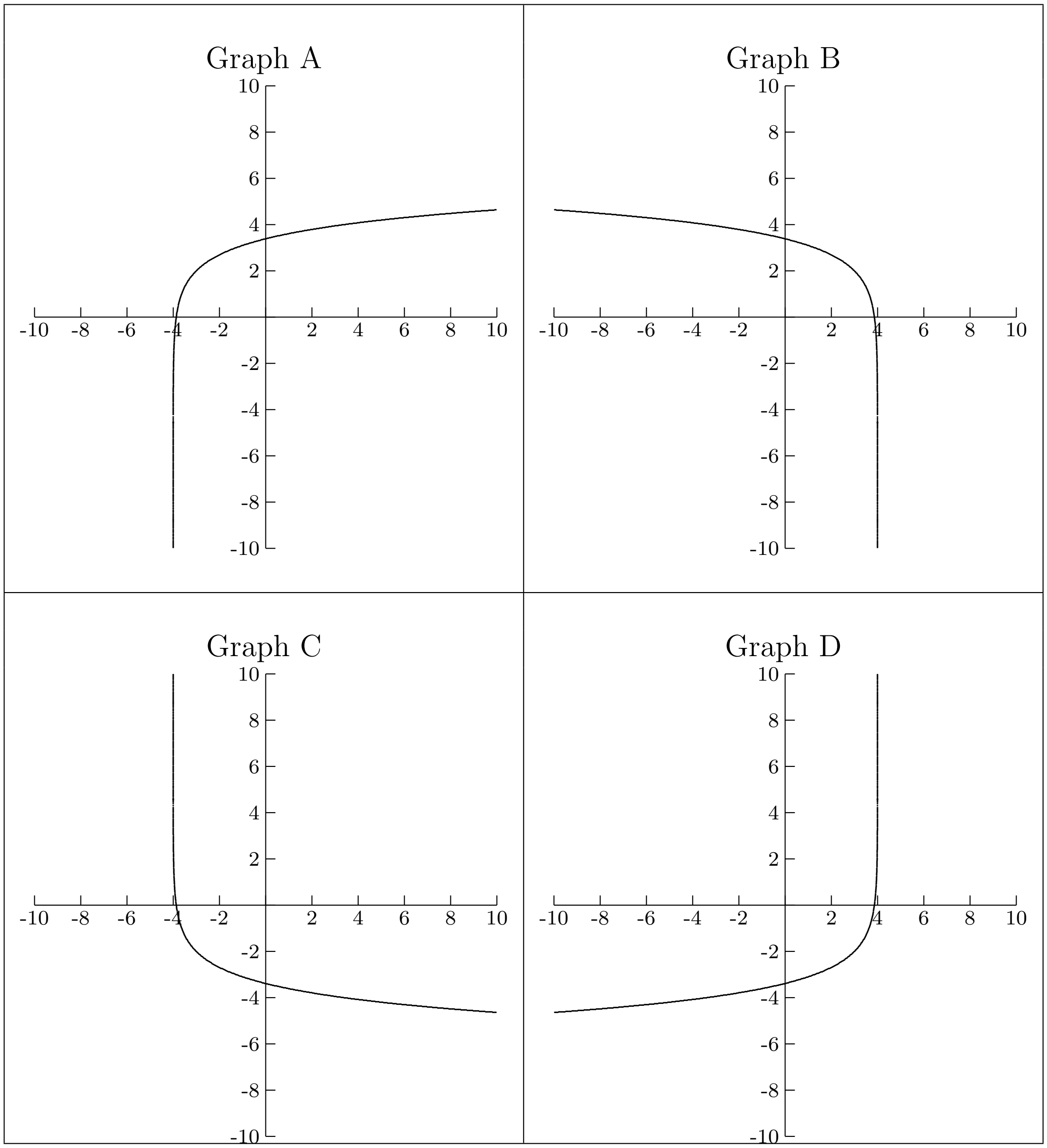
(c) $P(d) = -1.336f(x) + 84.862$

(d) $D(p) = -1336p + 84.862$

Solution

Use the regression feature of a graphing calculator to see that the answer is (a).

8. Use transformations of the graph of $y = \ln x$ to sketch the graph of $f(x) = 2 + \ln(4 - x)$.



Solution

The graph of $f(x) = 2 + \ln(4 - x) = 2 + \ln[-(x - 4)]$ is the graph of $y = \ln x$ reflected across the y-axis, shifted to the right 4 units and shifted up 2 units. The graph is in B.

9. Find k such that $f(x) = x^4 + kx^3 + 2$ has the factor $x - 1$.

   (a) 2
   (b) -2
   (c) 3
   (d) -3

   Solution

   We want the remainder of

$$\frac{x^4 + kx^3 + 0x^2 + 0x + 2}{x - 1}$$

   to be 0.

$$\begin{array}{r|rrrrr} 1 & 1 & k & 0 & 0 & 2 \\ & & 1 & k+1 & k+1 & \\ \hline & 1 & k+1 & k+1 & k+1 & k+3 \end{array}$$

   If the remainder is to be 0, then $k + 3 = 0$, giving us $k = -3$.

   We could, instead, find k using the Remainder Theorem. We want $f(1) = 0$:

   $0 = f(1) = 1^4 + k(1^3) + 2$ which gives us $0 = 1 + k + 2$ solving for k gives us $k = -3$.

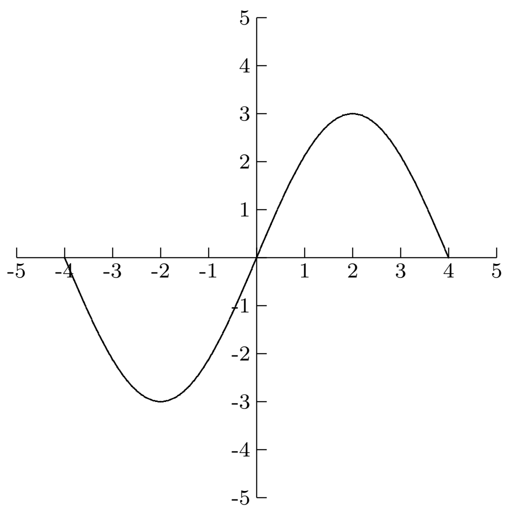
10. Find the horizontal and vertical asymptotes of the following rational function.

$$r(x) = \frac{7x + 1}{x - 8}$$

   (a) horizontal asymptote $y = 7$; vertical asymptote $x = 8$
   (b) horizontal asymptote $y = 7$; vertical asymptote $x = -8$
   (c) horizontal asymptote $y = 0$; vertical asymptote $x = -16$
   (d) horizontal asymptote $y = 0$; vertical asymptote $x = 8$
   (e) horizontal asymptote $y = 0$; vertical asymptote $x = -8$

   Solution

   We find the vertical asymptote(s) by locating the zero(s) of the denominator, so the vertical asymptote for the graph of this function is $x = 8$. The numerator and denominator have the same degree, so the horizontal asymptote is the line $y = \frac{\text{leading coefficient of the numerator}}{\text{leading coefficient of the denominator}}$; that is $y = 7$.



11. What is the domain of this function?

    (a) $[-4, 3]$

    (b) $[-4, 4]$

    (c) $[-3, 3]$

    (d) $[-3, 4]$

    Solution
    The graph extends horizontally from $x = -4$ to $x = 4$, so the domain is $[-4, 4]$.

12. For the given functions f and g, find the requested composite function value.

$$f(x) = \frac{x-6}{x}, \ g(x) = x^2 + 9, \quad \text{Find } (f \circ g)(-2).$$

    (a) $\frac{7}{13}$
    (b) 25
    (c) $\frac{145}{16}$
    (d) 13

    Solution

$$g(-2) = (-2)^2 + 9 = 13 \text{ and } f(13) = \frac{13-6}{13} = \frac{7}{13}$$

13. The function $A = A_0 e^{-0.00693x}$ models the weight (in pounds) of a particular radioactive material stored in a concrete vault, where x is the number of years since A_0 pounds of the material was put into the vault. If 500 pounds of the material is initially put into the vault, how many pounds will be left after 175 years?

    (a) 100 lbs
    (b) 425 lbs
    (c) 154 lbs
    (d) 149 lbs

    Solution

$$A = 500e^{-.00693(175)} \approx 149 \text{ lbs}$$

14. Solve the equation

$$\frac{3}{x+5} + \frac{2}{2x+1} = \frac{4}{x-2}$$

    (a) $\frac{47}{46}$
    (b) $-\frac{46}{47}$
    (c) $\frac{46}{47}$
    (d) $\frac{-47}{46}$

    Solution

    The LCD is $(x+5)(2x+1)(x-2)$. We begin by multiplying each side of the equation by the LCD.

$$\begin{aligned}
\frac{3}{x+5} + \frac{2}{2x+1} &= \frac{4}{x-2} \\
(x+5)(2x+1)(x-2)\left(\frac{3}{x+5} + \frac{2}{2x+1}\right) &= \frac{4}{x-2}(x+5)(2x+1)(x-2) \\
3(2x+1)(x-2) + 2(x+5)(x-2) &= 4(x+5)(2x+1) \\
3(2x^2-3x-2) + 2(x^2+3x-10) &= 4(2x^2+11x+5) \\
6x^2 - 9x - 6 + 2x^2 + 6x - 20 &= 8x^2 + 44x + 20 \\
8x^2 - 3x - 26 &= 8x^2 + 44x + 20 \\
-46 &= 47x \\
-\frac{46}{47} &= x
\end{aligned}$$

15. A motorboat maintained a constant speed of 18 miles per hour relative to the water in going 40 miles upstream and then returning. The total time for the trip was 4.5 hours. Use this information to find the speed of the current in miles per hour.

(a) 3 miles per hour

(b) 5 miles per hour

(c) 7 miles per hour

(d) 4 miles per hour

(e) 2 miles per hour

Solution

Let s represent the stream's speed. The boat's stream upstream is $18-s$ and is $18+s$ downstream. Let t_1 represent the time spent traveling upstream and t_2 the time spent traveling downstream. Then $t_1+t_2=4.5$. We need to make substitutions for t_1 and t_2 so that we have an equation with only one unknown. We use the model $D=rt$ for the trip upstream and again for the trip downstream. We then write the equation $t_1+t_2=4.5$ in terms of s.

$$\overset{D}{40}=(18\overset{r}{-}s)\overset{t}{t_1} \text{ gives us } t_1=\frac{40}{18-s}$$

and

$$40=(18+s)t_2 \text{ gives us } t_2=\frac{40}{18+s}$$

After making substitutions, we will solve the equation.

$$\begin{aligned} t_1+t_2 &= 4.5 \\ \frac{40}{18-s}+\frac{40}{18+s} &= 4.5 \\ (18-s)(18+s)\left(\frac{40}{18-s}+\frac{40}{18+s}\right) &= 4.5(18-s)(18+s) \quad \text{Multiply each side by the LCD.} \\ 40(18+s)+40(18-s) &= 4.5(18+s)(18-s) \\ 720+40s+720-40s &= 4.5(324-s^2) \\ 1440 &= 1458-4.5s^2 \\ 4.5s^2 &= 18 \\ s^2 &= 4 \\ s &= 2 \end{aligned}$$

The stream's speed is 2 mph.

16. Solve the system of equations.

$$\begin{cases} 2x + 4y - 2z = 0 \\ 3x + 5y = 1 \end{cases}$$

(a) $x = 2 + 7z$, and $y = -3z - 1$, where z is any real number

(b) $x = -2 - 7z$, and $y = -3z - 1$, where z is any real number

(c) $x = 2 - 5z$, and $y = 3z - 1$, where z is any real number

(d) $x = 1 - 5z$, and $y = 3z - \frac{1}{2}$, where z is any real number

Solution

All of the choices have x and y written in terms of z. We begin by replacing x in the second equation using the first equation.

$$\begin{aligned} 2x + 4y - 2z &= 0 & \\ x + 2y - z &= 0 & \text{Divide each side by 2.} \\ x &= z - 2y & \text{Solve for } x. \end{aligned}$$

The second equation becomes $3(z - 2y) + 5y = 1$ after substituting $z - 2y$ for x.

$$\begin{aligned} 3(z - 2y) + 5y &= 1 & \\ 3z - y &= 1 & \\ 3z - 1 &= y & \text{We now have the solution for } y \text{ in terms of } z. \end{aligned}$$

We now substitute $3z - 1$ for y in $x + 2y - z = 0$ and solving the equation for x.

$$\begin{aligned} x + 2(3z - 1) - z &= 0 & \\ x + 5z - 2 &= 0 & \\ x &= 2 - 5z & \text{We now have the solution for } x \text{ in terms of } z. \end{aligned}$$

Section 2–Free Response

17. Find $f^{-1}(x)$ (the inverse function) if $f(x) = \frac{1}{3x-2}$.

Solution

Begin by switching x and y. We then solve the equation for y.

$$\begin{aligned}
y &= \frac{1}{3x-2} \\
x &= \frac{1}{3y-2} \\
x(3y-2) &= 1 \\
3xy - 2x &= 1 \\
3xy &= 2x+1 && \text{Isolate the term containing } y. \\
y &= \frac{2x+1}{3x} \\
f^{-1}(x) &= \frac{2x+1}{3x}
\end{aligned}$$

18. Solve the equation.

$$x + \sqrt{x+44} = 12$$

Solution

$$\begin{aligned}
x + \sqrt{x+44} &= 12 && \text{Begin by isolating the radical.} \\
\sqrt{x+44} &= 12 - x \\
\left(\sqrt{x+44}\right)^2 &= (12-x)^2 && \text{Square each side.} \\
x + 44 &= 144 - 24x + x^2 && \text{This is a quadratic equation.} \\
0 &= x^2 - 25x + 100 \\
0 &= (x-5)(x-20) && \text{We solve by factoring.} \\
x &= 5,\ 20
\end{aligned}$$

Because we squared both sides of the equation, we must check our solutions in the original equation (squaring both sides of an equation can introduced *extraneous* solutions).

$$\begin{aligned}
x = 5: \quad & 5 + \sqrt{5+44} = 12? \\
& 5 + 7 = 12 && \text{This is true, so } x = 5 \text{ is a solution.} \\
x = 20: \quad & 20 + \sqrt{20+44} = 12? \\
& 20 + 8 = 12 && \text{This is not true, so } x = 20 \text{ is not a solution.}
\end{aligned}$$

19. Solve the equation by completing the square. If you solve the equation using any other method, you will receive **no** credit.

$$x^2 - 4x - 8 = 0$$

Solution

$$\begin{aligned}
x^2 - 4x - 8 &= 0 \\
x^2 - 4x &= 8 \\
x^2 - 4x + \left(\frac{-4}{2}\right)^2 &= 8 + \left(\frac{-4}{2}\right)^2 \\
x^2 - 4x + 4 &= 12 \\
(x-2)^2 &= 12 \\
x - 2 &= \pm\sqrt{12} = \pm 2\sqrt{3} \\
x &= 2 \pm 2\sqrt{3}
\end{aligned}$$

20. Suppose that the quantity supplied S and quantity demanded D of T-shirts at a concert are given by the following functions, where p is the price per T-shirt.
Solution

$$S(p) = -250 + 60p$$
$$D(p) = 1370 - 75p$$

Find the equilibrium price p (defined as the price at which quantity supplied equals quantity demanded).

$$\begin{aligned} -250 + 60p &= 1370 - 75p \\ 135p &= 1620 \\ p &= \frac{1620}{135} = 12 \end{aligned}$$

The equilibrium price is \$12 per shirt.

21. Solve the equation.

$$\log_2(7x+4) - \log_2(x-2) = 4$$

We begin by writing the left side of the equation as a single logarithm using the property $\log_a u - \log_a v = \log_a \frac{u}{v}$.

$$\begin{aligned} \log_2 \frac{7x+4}{x-2} &= 4 \\ 2^4 &= \frac{7x+4}{x-2} && \text{Rewrite the equation in exponential form.} \\ 16(x-2) &= 7x+4 && \text{Cross-multiply.} \\ 16x - 32 &= 7x + 4 \\ 9x &= 36 \\ x &= 4 \end{aligned}$$

Because we used a log property to write the equation, we must check our solution in the original equation.

$$\log_2(7 \cdot 4 + 4) - \log_2(4-2) = \log_2 32 - \log_2 2 = \log_2 2^5 - \log_2 2 = 5 - 1 = 4$$

The solution $x = 4$ is verified.

22. Graph the solution to the system of inequalities. (There is no work that you need to show.)

$$\begin{cases} x - y < 4 \\ y < -x^2 + 2x + 8 \end{cases}$$

Solution

We can find the vertex for the graph of $y = -x^2 + 2x + 8$ using completing the square.

$$\begin{aligned} y &= -x^2 + 2x + 8 \\ y &= -(x^2 - 2x) + 8 \\ y &= -(x^2 - 2x + 1) + 8 + 1 \\ y &= -(x - 1)^2 + 9 \qquad \text{The vertex is } (1, 9). \end{aligned}$$

Because the inequalities are strict, we use dashes to sketch the graphs of $y = -x^2+2x+8$ and $x - y = 4$.

We find the x-intercepts by factoring.

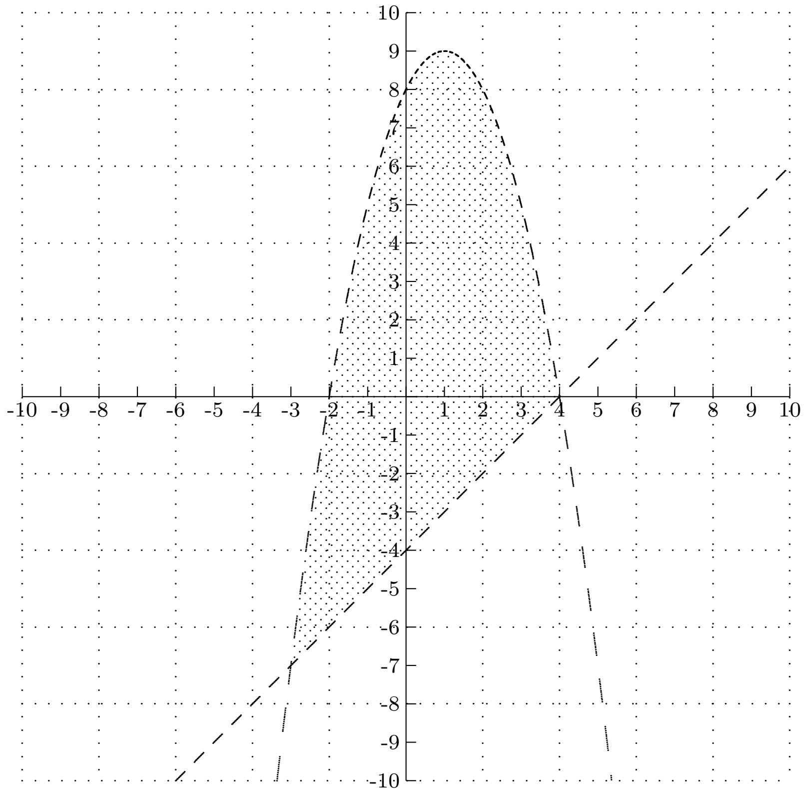
$$y = -x^2 + 2x + 8 = -(x^2 - 2x - 8) = -(x - 4)(x + 2)$$

The x-intercepts are $(4, 0)$ and $(-2, 0)$.

We can also find where the graphs intersect by setting the y-values equal and solving for x.

$$\begin{aligned} -x^2 + 2x + 8 &= x - 4 \\ 0 &= x^2 - x - 12 \\ 0 &= (x - 4)(x + 3) \qquad \text{The solutions are } x = 4 \text{ and } x = -3. \end{aligned}$$

This work shows that the graphs should intersect at $x = 4$ and $x = -3$. We shade under the parabola and above the line between $x = -3$ and $x = 4$.



-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 1 2 3 4 5 6 7 8 9 10
10 9 8 7 6 5 4 3 2 1 -1 -2 -3 -4 -5 -6 -7 -8 -9 -10

Cover Page for MATH 1100 Final Exam^A
Spring 2010

Name (Print) ____________________________

Student Number ______________________

Instructor Name ____________________________

Section Number ______________

May 11, 2010

This exam consists of two sections. The first section contains 16 multiple-choice questions. **Only answers recorded properly on the scantron with a number 2 pencil will be graded**, so carefully bubble in the answers on your scantron. Make sure your name, your instructor's name, and your section number are recorded on your scantron and in the spaces provided above. Each multiple-choice question is worth 4 points.

The second section of the exam consists of six free-response questions. You must show work to receive credit, and only work shown on the exam itself will be graded. Please transfer whatever work that you want graded from your scratch paper to the exam itself. Each free-response problem is worth 7 points. There is a total of 106 points on the test.

No notes are allowed on the exam and only approved calculators are allowed. You are not allowed to use any non-medical electronic devices, i.e. ear phones, plugs, etc. Cell phones must be TURNED OFF and PUT AWAY.

Do **NOT** write in the blanks below. They are for grade recording purposes only.

Problem 17 ______(7 points)

Problem 18 ______(7 points)

Problem 19 ______(7 points)

Problem 20 ______(7 points)

Problem 21 ______(7 points)

Problem 22 ______(7 points)

Multiple-Choice: ________ $\times 4 =$ ____________ (64 points)

MATH 1100 FINAL EXAM

Spring 2010

Multiple-Choice

Record your answers on a scantron. Make sure that erasures are cleanly made.

1. Determine algebraically whether the given function is even, odd, or neither.

$$f(x) = 4x + |3x|$$

   (a) The function is even.
   (b) The function is odd.
   (c) The function is neither.
   (d) There is not enough information to answer this question.

2. An experienced bank auditor can check a bank's deposits twice as fast as a new auditor. Working together, it takes the auditors 12 hours to do the job. How long would it take the experienced auditor working alone?

   (a) 24 hours
   (b) 18 hours
   (c) 36 hours
   (d) 12 hours

3. Solve the equation in the real number system.

$$3x^3 - 17x^2 + 18x + 8 = 0$$

   (a) $\{-\frac{1}{3},\ 2,\ 4\}$
   (b) $\{-\frac{4}{3},\ -1,\ -2\}$
   (c) $\{\frac{4}{3},\ -1,\ 2\}$
   (d) $\{\frac{1}{3},\ 2,\ -4\}$

4. The time it takes a satellite to complete an orbit around the earth varies directly as the radius of the orbit from the center of the earth and inversely as its orbital velocity. If a satellite completes and orbit 670 miles above the earth in 11 hours at a velocity of 21000 mph, how long would it take a satellite to complete an orbit if it is at 1530 miles above the earth at a velocity of 36000 mph? (Use 3960 miles as the radius of the earth.)

   (a) 4.99 hours
   (b) 25.12 hours
   (c) 7.61 hours
   (d) 8.12 hours

5. A vendor has learned that, by pricing hot dogs at \$1.75, sales will reach 107 hot dogs per day. Raising the price to \$2.75 will cause sales to fall to 67 hot dogs per day. Write a linear equation that relates the number of hot dogs sold per day to the price. Let y be the number of hot dogs the vendor sells at x dollars each.

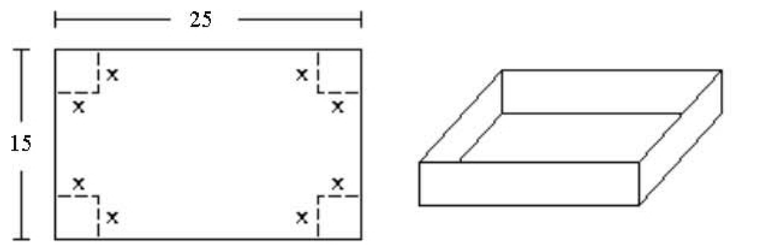
   (a) $y = 40x + 37$
   (b) $y = -\frac{1}{40}x + 107$
   (c) $y = -40x - 177$
   (d) $y = -40x + 177$

6. Determine where the function is increasing and where it is decreasing.

$$f(x) = -x^2 - 8x$$

   (a) The function is decreasing on the interval $(-\infty, -4)$ and the function is increasing on the interval $(-4, \infty)$
   (b) The function is decreasing on the interval $(-\infty, 4)$ and the function is increasing on the interval $(4, \infty)$
   (c) The function is increasing on the interval $(-\infty, -4)$ and the function is decreasing on the interval $(-4, \infty)$
   (d) The function is increasing on the interval $(-\infty, 4)$ and the function is decreasing on the interval $(4, \infty)$

7. A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 15 inches by 25 inches by cutting out equal squares of side x at each corner and then folding up the sides as in the figure. Express the volume, V as a function of x.



(a) $V(x) = (15 - x)(25 - x)$
(b) $V(x) = x(15 - 2x)(25 - 2x)$
(c) $V(x) = x(15 - x)(25 - x)$
(d) $V(x) = (15 - 2x)(25 - 2x)$

8. Solve the equation.

$$\log_9(x^2 - 8x) = 1$$

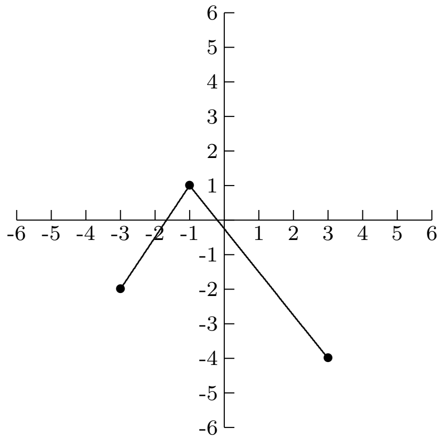
(a) $\{ 9 \}$
(b) $\{ -9, 1 \}$
(c) $\{ 9, -1 \}$
(d) $\{ 1 \}$

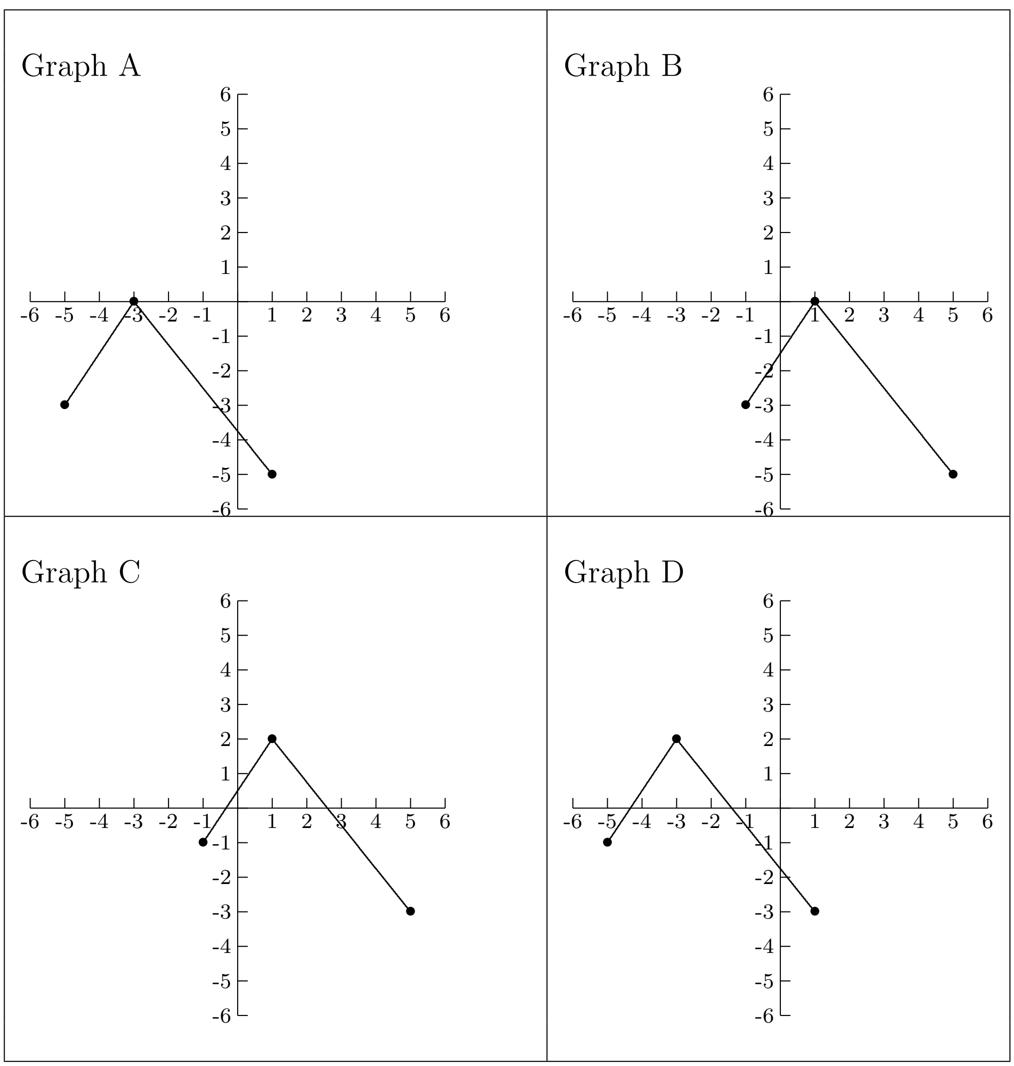
9. Use a graphing utility to find the equation of the line of best fit. Round to three decimal places, if necessary.

x	6	8	20	28	36
y	2	4	13	20	30

(a) $y = 0.897x + 3.790$
(b) $y = 0.897x - 3.790$
(c) $y = 1.102x + 4.391$
(d) $y = 1.102x - 4.391$

10. The graph of the function $y = f(x)$ is given below. Identify the graph of the function $y = f(x+2) - 1$.





11. Solve the inequality.

$$(x-3)(x-4)(x-6) < 0$$

(a) $(-\infty, 3)$ or $(4, 6)$

(b) $(6, \infty)$

(c) $(-\infty, 4)$

(d) $(3, 4)$ or $(6, \infty)$

12. A certain radioactive isotope decays as a rate of 0.1% annually. Determine the half-life of this isotope. Give your answer accurate to the nearest year.

(a) 693 years

(b) 301 years

(c) 7 years

(d) 347 years

13. Find the quadratic function with vertex $(4, 1)$ and whose graph goes through the point $(5, 3)$.

(a) $f(x) = -x^2 + 11x - 27$

(b) $f(x) = -4x^2 + 40x - 97$

(c) $f(x) = -2x^2 + 20x - 47$

(d) $f(x) = 2x^2 - 16x + 33$

14. Express as a single logarithm.

$$8\ln(x-6) - 7\ln x$$

(a) $\ln \frac{8(x-6)}{7x}$

(b) $\ln x^7(x-6)^8$

(c) $\ln 56x(x-6)$

(d) $\ln \frac{(x-6)^8}{x^7}$

15. Find functions f and g so that $f \circ g = H$.

$$H(x) = \frac{2}{\sqrt{3x+9}}$$

(a) $f(x) = \frac{2}{x}$; $g(x) = 3x+9$

(b) $f(x) = \sqrt{3x+9}$; $g(x) = 2$

(c) $f(x) = 2$; $g(x) = \sqrt{3x+9}$

(d) $f(x) = \frac{2}{\sqrt{x}}$; $g(x) = 3x+9$

16. In a town whose population is 3000, a disease creates an epidemic. The number of people, N, infected t days after the disease has begun is given by the function

$$N(t) = \frac{3000}{1 + 21.2e^{-0.54t}}.$$

Find the number of infected people after 10 days.

(a) 1000 people

(b) 2738 people

(c) 142 people

(d) 2340 people

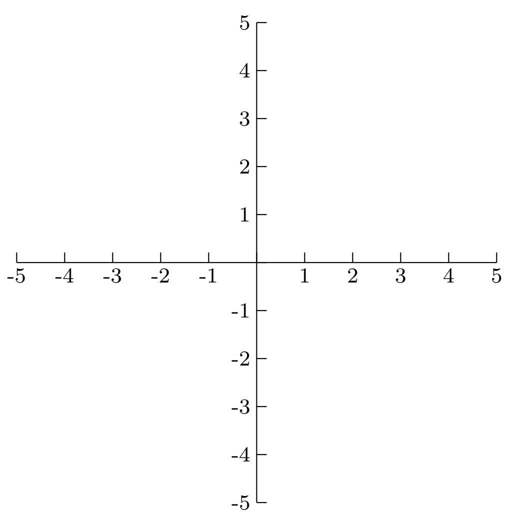
Free Response

Your work will be graded on this part of the test, so make your work neat and organized.

17. Find the equation of the secant line at $x = 2$ with $h = 1$ for the function. Write your answer in slope-intercept form, $y = mx + b$.

$$g(x) = 5x^2 - 6x + 6$$

18. Sketch the solution of the system. $f(x) = \begin{cases} y > x^2 \\ y < x + 2 \end{cases}$.



19. Let $f(x) = \sqrt[3]{x} - 4$. Find $f^{-1}(x)$.

20. Analyze the graph of the function.

$$r(x) = \frac{x^2 + x - 30}{x^2 - 2x - 24}$$

(a) What is the domain of the function? You may give your answer either as an inequality or inequalities or you may give your answer in interval notation.

(b) What are the intercepts?

The x-intercept(s) is/are __________________.

The y-intercept is __________________.

(c) What are the asymptotes?

The vertical asymptote(s) is/are __________________.

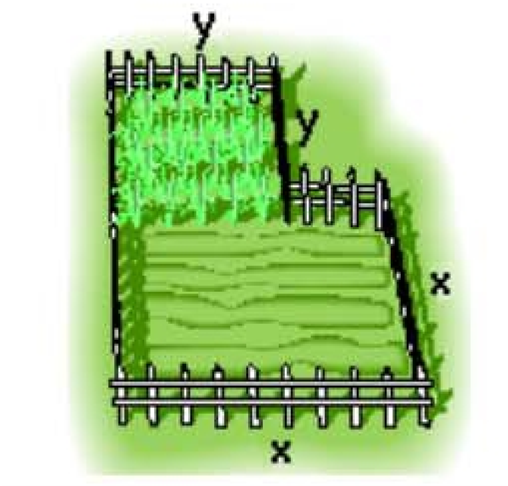
The horizontal asymptote is __________________.

21. Solve the system of equations.

$$\begin{cases} x - 3y + 4z & = 16 \\ 2x + y + z & = -3 \\ -2x + 3y - 3z & = -13 \end{cases}$$

$x =$ ________ $y =$ ________ $z =$ ________

22. A farmer has 200 feet of fencing available to enclose a 2000-square foot region in the shape of adjoining squares, with sides of length x and y. The big square has sides of length x and the small square has sides of length y. Find x and y.



$x =$ _______ $\qquad$ $y =$ _______

MATH 1100 FINAL EXAM

Spring 2010

1. Determine algebraically whether the given function is even, odd, or neither.

$$f(x) = 4x + |3x|$$

   (a) The function is even.
   (b) The function is odd.
   (c) The function is neither.
   (d) There is not enough information to answer this question.

   Solution

   Evaluate the function at $-x$ and compare to $f(x)$ and to $-f(x)$.

$$\begin{aligned} f(-x) &= 4(-x) + |3(-x)| \\ f(-x) &= -4x + |3x| \end{aligned}$$

   Because $f(-x) \neq f(x)$, f is not even. Because $f(-x) \neq -f(x)$, f is not odd.

2. An experienced bank auditor can check a bank's deposits twice as fast as a new auditor. Working together, it takes the auditors 12 hours to do the job. How long would it take the experienced auditor working alone?

   (a) 24 hours
   (b) 18 hours
   (c) 36 hours
   (d) 12 hours

   Solution

   Let t_1 = time required for the experienced auditor to check the bank's deposits and t_2 = the time required for the new auditor to check the deposits. This gives us

$$\frac{1}{t_1} + \frac{1}{t_2} = \frac{1}{12}$$

   Because the experienced auditor is twice as fast, $t_2 = 2t_1$. Making this substitution in the above gives us

$$\frac{1}{t_1} + \frac{1}{2t_1} = \frac{1}{12}$$

   Solve this for t_1. Begin by multiplying each side of the equation by the LCD, $12t_1$.

$$\begin{aligned} 12t_1\left(\frac{1}{t_1} + \frac{1}{2t_1}\right) &= 12t_1\left(\frac{1}{12}\right) \\ 12 + 6 &= t_1 \\ 18 &= t_1 \end{aligned}$$

   The experienced auditor would need 18 hours to work along to check the bank's deposits.

3. Solve the equation in the real number system.

$$3x^3 - 17x^2 + 18x + 8 = 0$$

(a) $\{-\frac{1}{3},\ 2,\ 4\}$

(b) $\{-\frac{4}{3},\ -1,\ -2\}$

(c) $\{\frac{4}{3},\ -1,\ 2\}$

(d) $\{\frac{1}{3},\ 2,\ -4\}$

Solution

According to the Rational Zeros Theorem, the candidates for the rational zeros are $\pm1,\ \pm2,\ \pm4,\ \pm8,\ \pm\frac{1}{3},\ \pm\frac{2}{3},\ \pm\frac{4}{3},\ \pm\frac{8}{3}$. Find one zero (either with a calculator, direct checking or using synthetic division). A zero is 2.

$$\begin{array}{r|rrrr} 2 & 3 & -17 & 18 & 8 \\ & & 6 & -22 & -8 \\ \hline & 3 & -11 & -4 & 0 \end{array}$$

We can find the other zeros by finding the zeros of the quotient, so we need to solve the following.

$$3x^2 - 11x - 4 = 0$$

Solve either by factoring, using the quadratic formula, or completing the square.

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(3)(-4)}}{2(3)} = \frac{11 \pm \sqrt{121 + 48}}{6} = \frac{11 \pm 13}{6} = -\frac{1}{3}, 4$$

The zeros are $2,\ -\frac{1}{3},\ 4$.

4. The time it takes a satellite to complete an orbit around the earth varies directly as the radius of the orbit from the center of the earth and inversely as its orbital velocity. If a satellite completes an orbit 670 miles above the earth in 11 hours at a velocity of 21000 mph, how long would it take a satellite to complete an orbit if it is at 1530 miles above the earth at a velocity of 36000 mph? (Use 3960 miles as the radius of the earth.)

   (a) 4.99 hours
   (b) 25.12 hours
   (c) 7.61 hours
   (d) 8.12 hours

   Solution
   Begin with the model $t = k\frac{r}{v}$, where t is the orbital time (in hours), v is the orbital velocity (in mph), and r is the distance (in miles) from the satellite to the center of the earth. First, find k.

$$\begin{aligned} 11 &= k\frac{670+3960}{21,000} \\ 231,000 &= 4630k \\ k &= \frac{23100}{463} \\ t &= \frac{23100}{463}\cdot\frac{1530+3960}{36,000} && \text{Now, find } t \text{ with the other information.} \\ t &\approx 7.61 \text{ hours} \end{aligned}$$

5. A vendor has learned that, by pricing hot dogs at \$1.75, sales will reach 107 hot dogs per day. Raising the price to \$2.75 will cause sales to fall to 67 hot dogs per day. Write a linear equation that relates the number of hot dogs sold per day to the price. Let y be the number of hot dogs the vendor sells at x dollars each.

   (a) $y = 40x + 37$
   (b) $y = -\frac{1}{40}x + 107$
   (c) $y = -40x - 177$
   (d) $y = -40x + 177$

   Solution
   In the linear equation, points are in the form (price, number sold), so we want the linear function whose graph contains the points $(2.75, 67)$ and $(1.75, 107)$.

$$\begin{aligned} m &= \frac{107-67}{1.75-2.75} = -40 \\ y - 67 &= -40(x-2.75) && \text{Use } m = -40 \text{ and the point } (2.75, 67). \\ y &= -40x + 177 \end{aligned}$$

6. Determine where the function is increasing and where it is decreasing.

$$f(x) = -x^2 - 8x$$

(a) The function is decreasing on the interval $(-\infty, -4)$ and the function is increasing on the interval $(-4, \infty)$

(b) The function is decreasing on the interval $(-\infty, 4)$ and the function is increasing on the interval $(4, \infty)$

(c) The function is increasing on the interval $(-\infty, -4)$ and the function is decreasing on the interval $(-4, \infty)$

(d) The function is increasing on the interval $(-\infty, 4)$ and the function is decreasing on the interval $(4, \infty)$

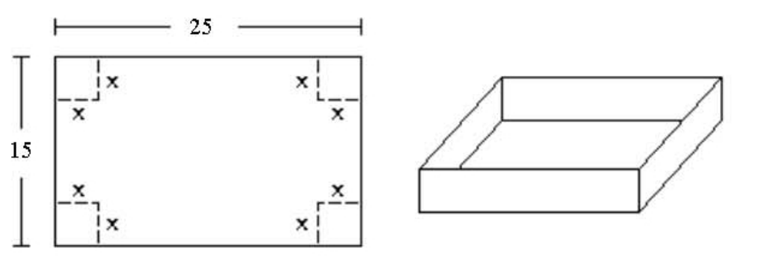
Solution

We begin by writing the function in standard form using completing the square.

$$\begin{aligned} f(x) &= -x^2 - 8x \\ f(x) &= -(x^2 + 8x + 16) + 16 \\ f(x) &= -(x + 4)^2 + 16 \end{aligned}$$

The graph is a parabola that opens down having vertex $(-4, 16)$. This means that the function is increasing on $(-\infty, -4)$ and decreasing on $(-4, \infty)$.

7. A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 15 inches by 25 inches by cutting out equal squares of side x at each corner and then folding up the sides as in the figure. Express the volume, V as a function of x.



(a) $V(x) = (15 - x)(25 - x)$

(b) $V(x) = x(15 - 2x)(25 - 2x)$

(c) $V(x) = x(15 - x)(25 - x)$

(d) $V(x) = (15 - 2x)(25 - 2x)$

Solution

The height of the box is x. Its length is $25 - 2x$, and its width is $15 - 2x$. The volume formula $V = LWH$ becomes

$$V = (25 - 2x)(15 - 2x)x$$

8. Solve the equation.

$$\log_9(x^2 - 8x) = 1$$

   (a) $\{ 9 \}$
   (b) $\{ -9, 1 \}$
   (c) $\{ 9, -1 \}$
   (d) $\{ 1 \}$

Solution

$$\begin{aligned} \log_9(x^2 - 8x) &= 1 \\ x^2 - 8x &= 9^1 \\ x^2 - 8x - 9 &= 0 \\ (x - 9)(x + 1) &= 0 \end{aligned}$$

The solutions are $x = 9$ and $x = -1$.

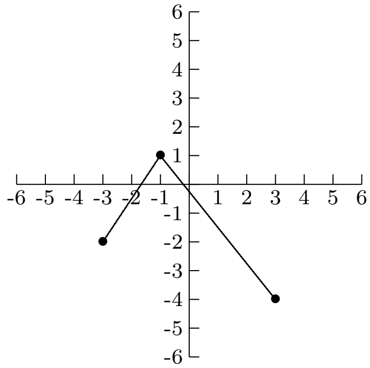
9. Use a graphing utility to find the equation of the line of best fit. Round to three decimal places, if necessary.

x	6	8	20	28	36
y	2	4	13	20	30

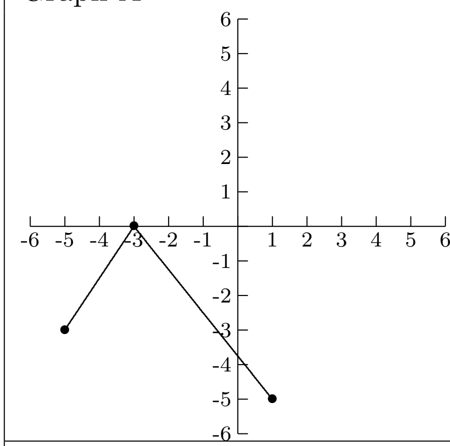
   (a) $y = 0.897x + 3.790$
   (b) $y = 0.897x - 3.790$
   (c) $y = 1.102x + 4.391$
   (d) $y = 1.102x - 4.391$

Solution
The answer is (b).

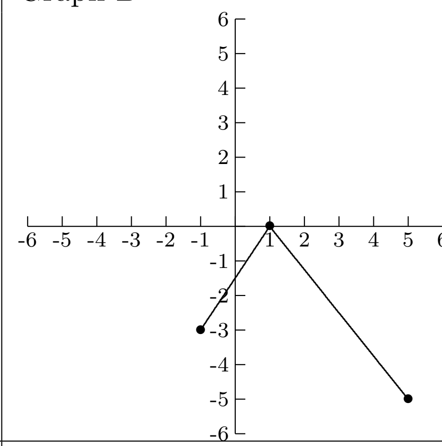
10. The graph of the function $y = f(x)$ is given below. Identify the graph of the function $y = f(x+2) - 1$.



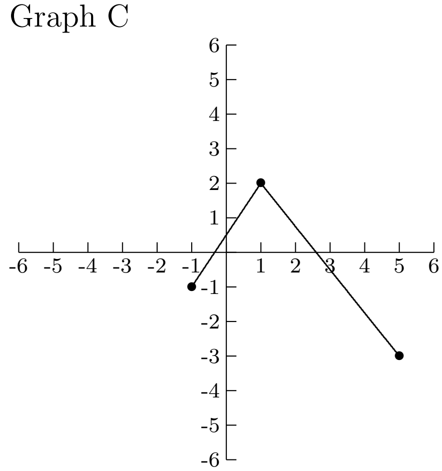
Graph A



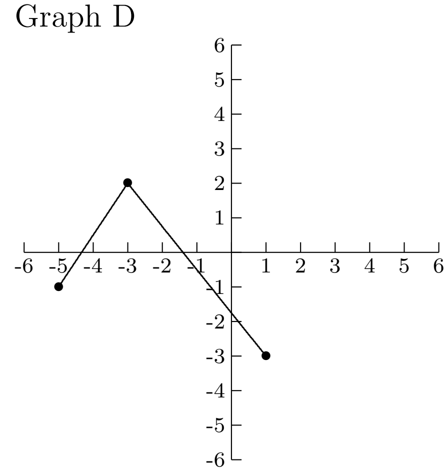
Graph B



Graph C



Graph D



Solution

The graph is shifted to the left 2 and down 1. The graph is A.

11. Solve the inequality.

$$(x-3)(x-4)(x-6) < 0$$

(a) $(-\infty, 3)$ or $(4, 6)$

(b) $(6, \infty)$

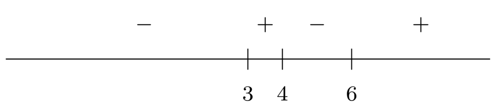
(c) $(-\infty, 4)$

(d) $(3, 4)$ or $(6, \infty)$

Solution

The zeros of the nonzero side of the inequality are 3, 4, and 6. Checking a value for x to the left of 3 gives a negative number. Checking a number between 3 and 4 gives a positive value. Checking a number between 4 and 6 gives a negative value. Checking a number to the right of 6 gives a positive value. (The verification steps are below.)

$$\begin{aligned}
x = 0: &\quad (0-3)(0-4)(0-6) = -72 \\
x = 3.5: &\quad (3.5-3)(3.5-4)(3.5-6) = 0.625 \\
x = 5: &\quad (5-3)(5-4)(5-6) = -2 \\
x = 7: &\quad (7-3)(7-4)(7-6) = 12
\end{aligned}$$



We want the intervals that give us negative values so the answer is (a).

12. A certain radioactive isotope decays as a rate of 0.1% annually. Determine the half-life of this isotope. Give your answer accurate to the nearest year.

    (a) 693 years
    (b) 301 years
    (c) 7 years
    (d) 347 years

    Solution

    Using the formula $h = \frac{\ln 2}{r}$ gives us

$$h = \frac{\ln 2}{0.001} \approx 693$$

    The half-life is about 693 years.

13. Find the quadratic function with vertex $(4, 1)$ and whose graph goes through the point $(5, 3)$.

    (a) $f(x) = -x^2 + 11x - 27$
    (b) $f(x) = -4x^2 + 40x - 97$
    (c) $f(x) = -2x^2 + 20x - 47$
    (d) $f(x) = 2x^2 - 16x + 33$

    Solution

    In $f(x) = a(x - h)^2 + k$, $h = 4$, $k = 1$, so $f(x) = a(x - 4)^2 + 1$. Find a by substituting 5 for x and 3 for $f(x)$.

$$\begin{aligned} 3 &= a(5 - 4)^2 + 1 \\ 3 &= a + 1 \\ a &= 2 \\ f(x) &= 2(x - 4)^2 + 1 \\ f(x) &= 2(x^2 - 8x + 16) + 1 \\ f(x) &= 2x^2 - 16x + 33 \end{aligned}$$

14. Express as a single logarithm.

$$8\ln(x-6) - 7\ln x$$

(a) $\ln \frac{8(x-6)}{7x}$

(b) $\ln x^7(x-6)^8$

(c) $\ln 56x(x-6)$

(d) $\ln \frac{(x-6)^8}{x^7}$

Solution

$$8\ln(x-6) - 7\ln x = \ln(x-6)^8 - \ln x^7 = \ln\left(\frac{(x-6)^8}{x^7}\right)$$

15. Find functions f and g so that $f \circ g = H$.

$$H(x) = \frac{2}{\sqrt{3x+9}}$$

(a) $f(x) = \frac{2}{x}$; $g(x) = 3x+9$

(b) $f(x) = \sqrt{3x+9}$; $g(x) = 2$

(c) $f(x) = 2$; $g(x) = \sqrt{3x+9}$

(d) $f(x) = \frac{2}{\sqrt{x}}$; $g(x) = 3x+9$

Solution

(d) is correct.

16. In a town whose population is 3000, a disease creates an epidemic. The number of people, N, infected t days after the disease has begun is given by the function

$$N(t) = \frac{3000}{1 + 21.2e^{-0.54t}}.$$

Find the number of infected people after 10 days.

(a) 1000 people

(b) 2738 people

(c) 142 people

(d) 2340 people

Solution

$$N(10) = \frac{3000}{1 + 21.2e^{-.54(10)}} \approx \frac{3000}{1.09575} \approx 2740$$

Free Response

Your work will be graded on this part of the test, so make your work neat and organized.

17. Find the equation of the secant line at $x = 2$ with $h = 1$ for the function. Write your answer in slope-intercept form, $y = mx + b$.

$$g(x) = 5x^2 - 6x + 6$$

Solution

The secant line goes through the points $(2, g(2))$ and $(2 + 1, g(2 + 1))$ which are the points $(2, 14)$ and $(3, 33)$.

$$m = \frac{33 - 14}{3 - 2} = 19$$

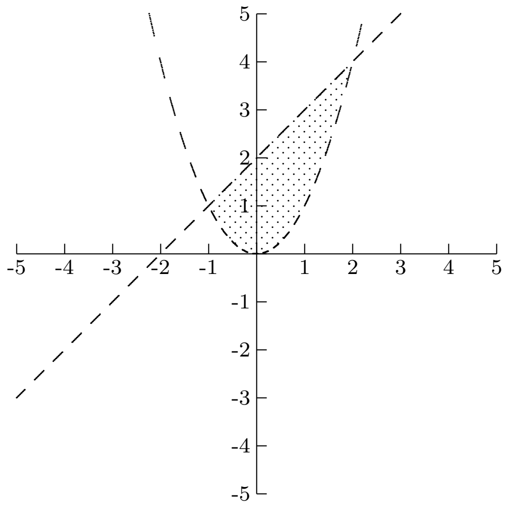
Using $m = 19$ and $(2, 14)$:

$$\begin{aligned} y - 14 &= 19(x - 2) \\ y - 14 &= 19x - 38 \\ y &= 19x - 24 \end{aligned}$$

18. Sketch the solution of the system. $f(x) = \begin{cases} y > x^2 \\ y < x + 2 \end{cases}$.

Solution

Use dashes to sketch the graphs of the functions. Shade above the parabola and below the line.



19. Let $f(x) = \sqrt[3]{x} - 4$. Find $f^{-1}(x)$.

Solution

Interchange the roles of x and y and then solve for y.

$$\begin{aligned} x &= \sqrt[3]{y} - 4 \\ x + 4 &= \sqrt[3]{y} \\ (x+4)^3 &= y \\ f^{-1}(x) &= (x+4)^3 \end{aligned}$$

20. Analyze the graph of the function.

$$r(x) = \frac{x^2 + x - 30}{x^2 - 2x - 24}$$

Solution

We begin by factoring the numerator and denominator.

$$r(x) = \frac{x^2 + x - 30}{x^2 - 2x - 24} = \frac{(x+6)(x-5)}{(x-6)(x+4)}$$

(a) What is the domain of the function? You may give your answer either as an inequality or inequalities or you may give your answer in interval notation.
Solution

$x \neq 6, -4$

(b) What are the intercepts?
The x-intercept(s) is/are _________________.

Solution

$(-6, 0)$ and $(5, 0)$

The y-intercept is _________________.
Solution

$(0, \frac{-30}{-24}) = (0, \frac{5}{4})$

(c) What are the asymptotes?

The vertical asymptote(s) is/are _________________.
Solution

$x = 6$ and $x = -4$

The horizontal asymptote is _________________.
Solution

$y = 1$

21. Solve the system of equations.

$$\begin{cases} x - 3y + 4z & = 16 \\ 2x + y + z & = -3 \\ -2x + 3y - 3z & = -13 \end{cases}$$

$x =$ ______ $y =$ ______ $z =$ ______

Solution

Eliminate y in two equations (or eliminate x or eliminate z) and then solve the 2×2 system. We begin by eliminating y by adding the first equation to 3 times the second equation (that is, multiply both sides of the second equation by 3.)

$$\begin{array}{rcrcrcr} x & - & 3y & + & 4z & = & 16 \\ 6x & + & 3y & + & 3z & = & -9 \\ \hline 7x & & & + & 7z & = & 7 \\ x & & & + & z & = & 1(*) \end{array}$$

Now, multiply the second equation by -3 and add to the third equation.

$$\begin{array}{rcrcrcr} -6x & - & 3y & - & 3z & = & 9 \\ -2x & + & 3y & - & 3z & = & -13 \\ \hline -8x & & & - & 6z & = & -4 \\ 4x & & & + & 3z & = & 2 \end{array}$$

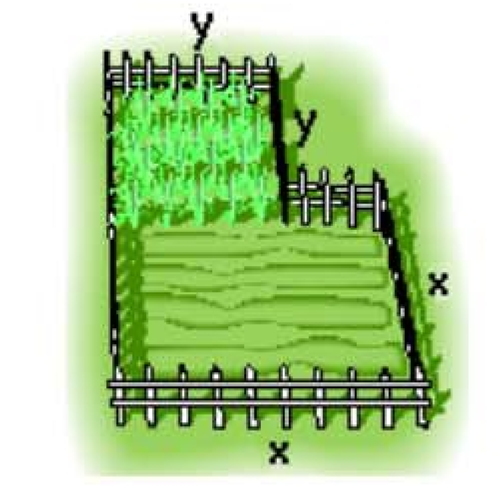
From (*), substitute $x = 1 - z$ in the above and solve for z.

$$\begin{aligned} 4(1-z) + 3z &= 2 \\ 4 - 4z + 3z &= 2 \\ z &= 2 \\ x &= 1 - 2 = -1 \qquad \text{Use the fact that } x = 1 - z. \end{aligned}$$

Now that we have two of the three unknowns, use any of the original equations to find y. We use the second equation.

$$\begin{aligned} 2(-1) + y + 2 &= -3 \\ y &= -3 \end{aligned}$$

22. A farmer has 200 feet of fencing available to enclose a 2000-square foot region in the shape of adjoining squares, with sides of length x and y. The big square has sides of length x and the small square has sides of length y. Find x and y.



$x =$ ______ $y =$ ______

Solution

The total enclosed area is $x^2 + y^2 = 2000$. Eliminate either x or y with the fact that the length of fencing is 200 feet. This fact gives us the equation

$$y + y + y + (x - y) + x + x + x = 200$$
$$2y + 4x = 200 \qquad \text{Now, solve for } y.$$
$$y = 100 - 2x$$

Substitute $100 - 2x$ for y in $x^2 + y^2 = 2000$ and solve for x.

$$x^2 + (100 - 2x)^2 = 2000$$
$$x^2 + 10,000 - 400x + 4x^2 = 2000$$
$$5x^2 - 400x + 8000 = 0$$
$$x^2 - 80x + 1600 = 0$$
$$(x - 40)^2 = 0$$
$$x - 40 = 0$$
$$x = 40$$
$$y = 100 - 2x = 100 - 2(40) = 20$$

Cover Page for MATH 1100 Final Exam^A
Fall 2010

Name (Print) ________________________________

Student Number ________________________

Instructor Name ________________________________

Section Number ________________

Time: 4:00-6:00

December 14, 2010

This exam consists of two sections. The first section contains 16 multiple-choice questions. **Only answers recorded properly on the scantron with a number 2 pencil will be graded**, so carefully bubble in the answers on your scantron. Make sure your name, your instructor's name, and your section number are recorded on your scantron and in the spaces provided above. Each multiple-choice question is worth 4 points.

The second section of the exam consists of six free-response questions. You must show work to receive credit, and only work shown on the exam itself will be graded. Please transfer whatever work that you want graded from your scratch paper to the exam itself. Each free-response problem is worth 7 points. There is a total of 106 points on the test.

No notes are allowed on the exam and only approved calculators are allowed. You are not allowed to use any non-medical electronic devices, i.e. ear phones, plugs, etc. Cell phones must be TURNED OFF and PUT AWAY. Do not share any materials during the exam, including but not limited to, calculators, pencils, erasers, etc. Also, please remove your hat.

Do **NOT** write in the blanks below. They are for grade recording purposes only.

Problem 17 ______(7 points)

Problem 18 ______(7 points)

Problem 19 ______(7 points)

Problem 20 ______(7 points)

Problem 21 ______(7 points)

Problem 22 ______(7 points)

Multiple-Choice: ________ $\times 4 =$ _____________ (64 points)

MATH 1100A FINAL EXAM

Fall 2010

Multiple-Choice

Record your answers on a scantron. Make sure that erasures are cleanly made.

1. A radiator in a certain make of a car needs to contain 20 liters of 40% antifreeze. The radiator now contains 20 liters of 20% antifreeze. How many liters of this solution must be drained and replaced with 100% antifreeze to get the desired strength?

   (a) 8 L
   (b) 10 L
   (c) 5 L
   (d) 9.2 L

2. Solve the inequality. Express your answer using interval notation.

$$|5 - 7x| > 9$$

   (a) $\left(-\infty, -\frac{4}{7}\right)$ or $\left(\frac{4}{7}, \infty\right)$
   (b) $\left(-\infty, -\frac{4}{7}\right)$ or $(2, \infty)$
   (c) $\left(\frac{4}{7}, -2\right)$
   (d) $\left(-\frac{4}{7}, 2\right)$

3. A vendor has learned that by pricing hot dogs at \$1.75, sales will reach 107 hot dogs per day. Raising the price to \$2.75 will cause the sales to fall to 67 hot dogs per day. Write a linear equation that relates the number of hot dogs sold per day to the price. Let y be the number of hot dogs the vendor sells at x dollars each.

   (a) $y = -\frac{1}{40}x + 107$
   (b) $y = -\frac{1}{40}x - 37$
   (c) $y = -40x - 177$
   (d) $y = -40x + 177$

4. A small manufacturing firm collected the following data on advertising expenditures (in thousands of dollars) and total revenue (in thousands of dollars). Use a graphing utility to plot the data and find the quadratic function of best fit. Round any decimals to three places.

Advertising (x)	25	28	31	32	34	39	40	45
Total Revenue (R)	6430	6432	6434	6434	6434	6431	6432	6420

(a) $R(x) = -0.024x^2 + 7.135x + 6209.125$

(b) $R(x) = -0.312x^2 + 2.633x + 6128.528$

(c) $R(x) = -0.015x^2 + 4.523x + 6123.527$

(d) $R(x) = -0.091x^2 + 5.952x + 6337.167$

5. Find the average rate of change of the function $f(x) = x^3 + x^2 - 5$ from $x = 2$ to $x = 3$.

(a) -8

(b) -16

(c) 16

(d) 24

6. If $f(x) = 2x - 1$ and $g(x) = 4x + 8$, then $g(f(x)) =$

(a) $8x + 4$

(b) $16x - 8$

(c) $8x + 15$

(d) $16x + 7$

7. Find the exact value of the logarithmic expression.

$$\log_9 \sqrt{9}$$

(a) $\frac{1}{9}$

(b) -1

(c) $\frac{1}{2}$

(d) 9

8. Find the inverse of the function.

$$\{(-3,4),\ (-1,5),\ (0,2),\ (2,6),\ (5,7)\}$$

(a) $\{(-3,-4),\ (-1,-5),\ (0,-2),\ (2,-6),\ (5,-7)\}$
(b) $\{(4,-3),\ (5,-1),\ (2,0),\ (6,2),\ (7,5)\}$
(c) $\{(3,4),\ (1,5),\ (0,2),\ (-2,6),\ (-5,7)\}$
(d) $\{(3,-4),\ (1,-5),\ (0,-2),\ (-2,-6),\ (-5,-7)\}$

9. Solve the equation.

$$\log_2(3x-2) - \log_2(x-5) = 4$$

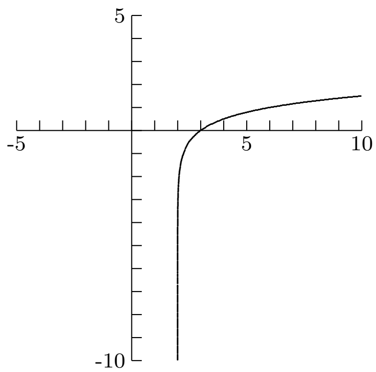
(a) $\{18\}$
(b) $\{\frac{3}{13}\}$
(c) $\{\frac{38}{5}\}$
(d) $\{6\}$

10. The function $f(x) = 800(0.50)^{x/50}$ models the amount in pounds of a particular radioactive material stored in a concrete vault, where x is the number of years since the material was put into the vault. Find the amount of radioactive material in the vault after 140 years. Round to the nearest whole number.

(a) 126 pounds
(b) 115 pounds
(c) 625 pounds
(d) 143 pounds

11. The graph of a logarithmic function is shown below. Select the function that matches the graph.

    (a) $y = \log_4(x + 2)$
    (b) $y = \log_4(x - 2)$
    (c) $y = 2 - \log_4 x$
    (d) $y = -2 + \log_4 x$



12. A ceramics workshop makes wreaths, trees, and sleighs for sale at Christmas. A wreath takes 3 hours to prepare, 2 hours to paint, and 8 hours to fire. A tree takes 14 hours to prepare, 3 hours to paint, and 4 hours to fire. A sleigh takes 4 hours to prepare, 17 hours to paint, and 7 hours to fire. If the workshop has 106 hours for preparation, 82 hours for painting, and 105 hours for firing, how many of each item can be made?

    (a) 6 wreaths, 6 trees, 7 sleighs
    (b) 9 wreaths, 5 trees, 4 sleighs
    (c) 9 wreaths, 6 trees, 4 sleighs
    (d) 8 wreaths, 5 trees, 3 sleighs

13. A man is planting a section of garden with tomatoes and cucumbers. The available area of the section is 130 square feet. He wants the area planted with tomatoes to be more than 25% of the area planted with cucumbers. Write a system of inequalities to describe the situation. Let x be the amount to be planted in tomatoes and y be the amount to be planted in cucumbers.

(a) $\begin{cases} x + y \leq 130 \\ x > 25y \\ x \geq 0 \\ y \geq 0 \end{cases}$

(b) $\begin{cases} x + y \geq 130 \\ x > 25y \\ x \geq 0 \\ y \geq 0 \end{cases}$

(c) $\begin{cases} x + y \leq 130 \\ x > 0.25y \\ x \geq 0 \\ y \geq 0 \end{cases}$

(d) $\begin{cases} x + y \leq 130 \\ x \leq 0.25y \\ x \geq 0 \\ y \geq 0 \end{cases}$

14. Solve the system of equations. If the system has no solution, say that it is inconsistent.

$$\begin{cases} 2x + y = 3 \\ -4x - 2y = -6 \end{cases}$$

(a) $y = -2x + 3$, where x is any real number

(b) $y = 2x + 3$, where x is any real number

(c) $x = -2y + 3$, where y is any real number

(d) The system is inconsistent.

15. Find the real zeros for the function

$$P(x) = x^3 - x^2 - 17x + 20.$$

(a) $4,\ \frac{3 \pm \sqrt{29}}{2}$

(b) $4,\ 3 \pm \frac{\sqrt{29}}{2}$

(c) $4,\ \frac{-3 \pm \sqrt{29}}{2}$

(d) $4,\ -3 \pm \frac{\sqrt{29}}{2}$

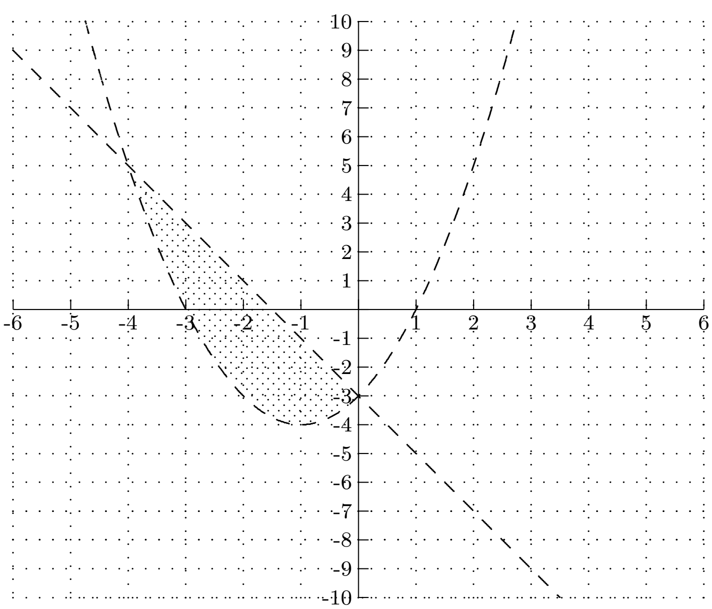
16. The graph below is the graph of which system in inequalities?

(a) $\begin{cases} y < x^2 + 2x - 3 \\ y > -2x - 3 \end{cases}$

(b) $\begin{cases} y < x^2 + 2x - 3 \\ y < -2x - 3 \end{cases}$

(c) $\begin{cases} y > x^2 + 2x - 3 \\ y > -2x - 3 \end{cases}$

(d) $\begin{cases} y > x^2 + 2x - 3 \\ y < -2x - 3 \end{cases}$



Free Response

Your work will be graded on this part of the test, so make your work neat and organized. No credit will be given for correct answers without the correct supporting work.

17. \$5000 is deposited into an account earning $7\frac{1}{2}\%$ annual interest, compounded monthly. Find the accumulated amount if the money is left in the account for 6 years. Please give your answer accurate to the nearest dollar. (Recall: $A = P(1 + \frac{r}{n})^{nt}$.)

18. Solve the following for x. A decimal approximation will receive no credit.

$$\frac{3x-2}{2x+4} = \frac{5}{8}$$

19. The graph of a function f is given below. Give your answers using interval notation. (It is not necessary to show your work on this problem.)

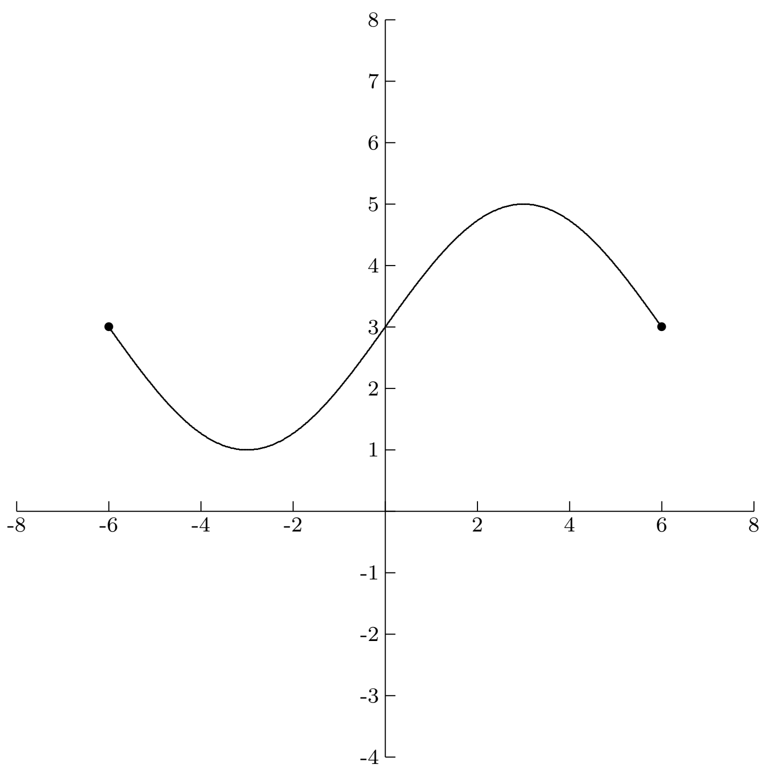
   (a) The domain is ________________.

   (b) The range is ________________.

   (c) Where is this function increasing and where is it decreasing?

      i. The function is increasing on the interval(s) _____________.

      ii. The function is decreasing on the interval(s) _____________.



20. Consider the rational function

$$R(x) = \frac{x^2 - 4}{x^2 - 9}.$$

(Remember, asymptotes are lines. Give vertical lines and horizontal lines in their proper form to earn full credit for the problems.) It is not necessary to show your work on this problem.

(a) What are the intercepts (both x- and y-intercepts)? (Please give your answers as points, that is, as ordered pairs.)

(b) What is/are the vertical asymptotes?

(c) What is the horizontal asymptote?

21. Determine the point of intersection of the following equations algebraically. You can use your graphing calculator to verify your solutions.

$$\begin{cases} y = x + 2 \\ y = -2x - 1 \end{cases}$$

22. Let $f(x) = x^2 - 6x + 8$.
Please show your complete work below.

(a) What is the vertex? ______________

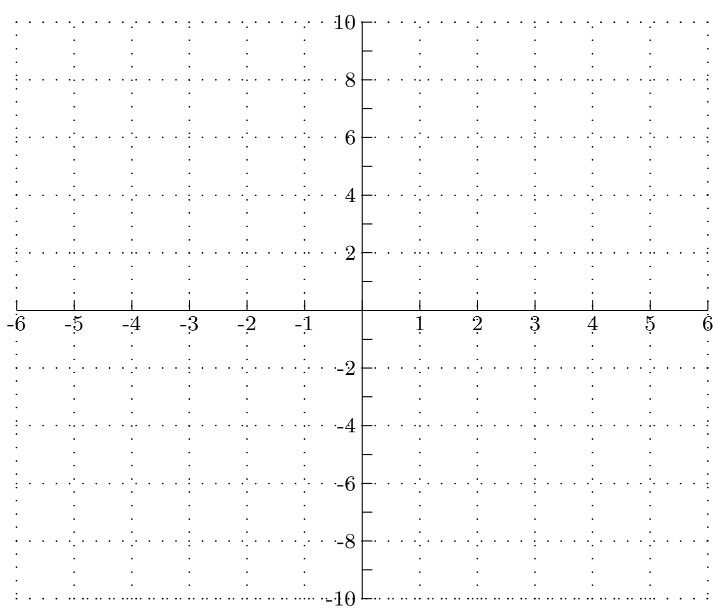
(b) What is the y-intercept?______________

(c) What are the x-intercepts? ______________

(d) What is the axis of symmetry (remember, this is a vertical line, so give your answer in the form of a vertical line.)

____

(e) Use your answers in (a)-(d) to sketch the graph.______________



MATH 1100A FINAL EXAM

Fall 2010

Multiple-Choice

Record your answers on a scantron. Make sure that erasures are cleanly made.

1. A radiator in a certain make of a car needs to contain 20 liters of 40% antifreeze. The radiator now contains 20 liters of 20% antifreeze. How many liters of this solution must be drained and replaced with 100% antifreeze to get the desired strength?

   (a) 8 L

   (b) 10 L

   (c) 5 L

   (d) 9.2 L

   Solution

   Let x represent the amount to be drained and replaced. This gives us the equation $\overset{\text{Drained}}{-0.20x} + \overset{\text{Replaced}}{1.00x} = 20(0.40)$. The solution to this equation is $x = 10$, so 10 liters should be drained and replaced so that the radiator contains 20 liters of a 20% solution of antifreeze.

2. Solve the inequality. Express your answer using interval notation.

$$|5 - 7x| > 9$$

   (a) $\left(-\infty, -\frac{4}{7}\right)$ or $\left(\frac{4}{7}, \infty\right)$

   (b) $\left(-\infty, -\frac{4}{7}\right)$ or $(2, \infty)$

   (c) $\left(\frac{4}{7}, -2\right)$

   (d) $\left(-\frac{4}{7}, 2\right)$

   Solution

   Solve the inequalities $5 - 7x < -9$ or $5 - 7x > 9$.

$$\begin{aligned} 5 - 7x &< -9 \quad \text{or } 5 - 7x > 9 \\ -7x &< -14 \text{ or } -7x > 4 \\ x &> 2 \qquad \text{or } x < -\frac{4}{7} \end{aligned}$$

   The answer is (b).

3. A vendor has learned that by pricing hot dogs at \$1.75, sales will reach 107 hot dogs per day. Raising the price to \$2.75 will cause the sales to fall to 67 hot dogs per day. Write a linear equation that relates the number of hot dogs sold per day to the price. Let y be the number of hot dogs the vendor sells at x dollars each.

   (a) $y = -\frac{1}{40}x + 107$
   (b) $y = -\frac{1}{40}x - 37$
   (c) $y = -40x - 177$
   (d) $y = -40x + 177$

   Solution

   We want the line containing the points $(1.75, 107)$ and $(2.75, 67)$.

$$m = \frac{107 - 67}{1.75 - 2.75} = -40$$

$$\begin{aligned} y &= -40x + b \\ 107 &= -40(1.75) + b && \text{Substitute the coordinates of one point to find } b. \\ 177 &= b \\ y &= -40x + 177 \end{aligned}$$

4. A small manufacturing firm collected the following data on advertising expenditures (in thousands of dollars) and total revenue (in thousands of dollars). Use a graphing utility to plot the data and find the quadratic function of best fit. Round any decimals to three places.

Advertising (x)	25	28	31	32	34	39	40	45
Total Revenue (R)	6430	6432	6434	6434	6434	6431	6432	6420

   (a) $R(x) = -0.024x^2 + 7.135x + 6209.125$
   (b) $R(x) = -0.312x^2 + 2.633x + 6128.528$
   (c) $R(x) = -0.015x^2 + 4.523x + 6123.527$
   (d) $R(x) = -0.091x^2 + 5.952x + 6337.167$

   Solution

   Use a calculator to find the answer, which is (d).

5. Find the average rate of change of the function $f(x) = x^3 + x^2 - 5$ from $x = 2$ to $x = 3$.

(a) -8
(b) -16
(c) 16
(d) 24

Solution

$f(2) = 2^3 + 2^2 - 5 = 7$ and $f(3) = 3^3 + 3^2 - 5 = 31$.

Solution

$$\text{Average rate of change } = \frac{f(3) - f(2)}{3 - 2} = \frac{31 - 7}{3 - 2} = 24$$

6. If $f(x) = 2x - 1$ and $g(x) = 4x + 8$, then $g(f(x)) =$

(a) $8x + 4$
(b) $16x - 8$
(c) $8x + 15$
(d) $16x + 7$

Solution

$$g(f(x)) = g(2x - 1) = 4(2x - 1) + 8 = 8x + 4$$

7. Find the exact value of the logarithmic expression.

$$\log_9 \sqrt{9}$$

(a) $\frac{1}{9}$
(b) -1
(c) $\frac{1}{2}$
(d) 9

Solution

$$\log_9 \sqrt{9} = \log_9 9^{1/2} = \frac{1}{2}$$

8. Find the inverse of the function.

$$\{(-3,4),\ (-1,5),\ (0,2),\ (2,6),\ (5,7)\}$$

(a) $\{(-3,-4),\ (-1,-5),\ (0,-2),\ (2,-6),\ (5,-7)\}$

(b) $\{(4,-3),\ (5,-1),\ (2,0),\ (6,2),\ (7,5)\}$

(c) $\{(3,4),\ (1,5),\ (0,2),\ (-2,6),\ (-5,7)\}$

(d) $\{(3,-4),\ (1,-5),\ (0,-2),\ (-2,-6),\ (-5,-7)\}$

Solution

The x- and y-coordinates for a function and its inverse are reversed, so the answer is (b).

9. Solve the equation.

$$\log_2(3x-2) - \log_2(x-5) = 4$$

(a) $\{18\}$

(b) $\{\frac{3}{13}\}$

(c) $\{\frac{38}{5}\}$

(d) $\{6\}$

Solution

$$\begin{aligned}
\log_2(3x-2) - \log_2(x-5) &= 4 \\
\log_2 \frac{3x-2}{x-5} &= 4 \\
2^4 &= \frac{3x-2}{x-5} \\
16(x-5) &= 3x-2 \\
16x-80 &= 3x-2 \\
78 &= 13x \\
6 &= x
\end{aligned}$$

10. The function $f(x) = 800(0.50)^{x/50}$ models the amount in pounds of a particular radioactive material stored in a concrete vault, where x is the number of years since the material was put into the vault. Find the amount of radioactive material in the vault after 140 years. Round to the nearest whole number.

    (a) 126 pounds
    (b) 115 pounds
    (c) 625 pounds
    (d) 143 pounds

    Solution

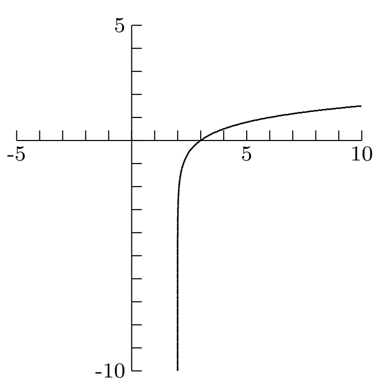
$$f(140) = 800\,(0.50)^{140/50} \approx 114.87$$

11. The graph of a logarithmic function is shown below. Select the function that matches the graph.

    (a) $y = \log_4(x + 2)$
    (b) $y = \log_4(x - 2)$
    (c) $y = 2 - \log_4 x$
    (d) $y = -2 + \log_4 x$

    Solution

    Because the base is more than 1 (the base is 4), the graph is a transformation of the function $y = \log_a x$, where $a > 1$. The graph below is the graph of $y = \log_a x$ shifted to the right 2 units, so the function is in (b).



12. A ceramics workshop makes wreaths, trees, and sleighs for sale at Christmas. A wreath takes 3 hours to prepare, 2 hours to paint, and 8 hours to fire. A tree takes 14 hours to prepare, 3 hours to paint, and 4 hours to fire. A sleigh takes 4 hours to prepare, 17 hours to paint, and 7 hours to fire. If the workshop has 106 hours for preparation, 82 hours for painting, and 105 hours for firing, how many of each item can be made?

    (a) 6 wreaths, 6 trees, 7 sleighs
    (b) 9 wreaths, 5 trees, 4 sleighs
    (c) 9 wreaths, 6 trees, 4 sleighs
    (d) 8 wreaths, 5 trees, 3 sleighs

Solution

Let x represent the number of wreaths; y, the number of trees, and z, the number of sleighs.

$$\begin{aligned} \text{Prepare} \quad & 3x + 14y + 4z = 106 \\ \text{Paint} \quad & 2x + 3y + 17z = 82 \\ \text{Fire} \quad & 8x + 4y + 7z = 105 \end{aligned}$$

There are numerous ways we can solve this system. Here, we will use the second equation to solve for x so that we can eliminate x from the first and third equations, giving us a 2×2 system to solve.

$$x = 41 - \frac{3}{2}y - \frac{17}{2}z$$

Now for the substitution:

$$3\left(41 - \frac{3}{2}y - \frac{17}{2}z\right) + 14y + 4z = 106, \text{ gives us } \frac{19}{2}y - \frac{43}{2}z = -17, \text{ or } 19y - 43z = -34$$

and

$$8\left(41 - \frac{3}{2}y - \frac{17}{2}z\right) + 4y + 7z = 105, \text{ gives us } -8y - 61z = -223$$

The solution to the system (you can use elimination by addition or by substitution or with matrices and a graphing calculator)

$$\begin{cases} 19y - 43z = -34 \\ -8y - 61z = -223 \end{cases}$$

is $y = 5$ and $z = 3$. We can find x using any equation having all three variables.

$$x = 41 - \frac{3}{2}y - \frac{17}{2}z = 41 - \frac{3}{2}(5) - \frac{17}{2}(3) = 8$$

The answer is (d).

13. A man is planting a section of garden with tomatoes and cucumbers. The available area of the section is 130 square feet. He wants the area planted with tomatoes to be more than 25% of the area planted with cucumbers. Write a system of inequalities to describe the situation. Let x be the amount to be planted in tomatoes and y be the amount to be planted in cucumbers.

(a) $\begin{cases} x + y \leq 130 \\ x > 25y \\ x \geq 0 \\ y \geq 0 \end{cases}$

(b) $\begin{cases} x + y \geq 130 \\ x > 25y \\ x \geq 0 \\ y \geq 0 \end{cases}$

(c) $\begin{cases} x + y \leq 130 \\ x > 0.25y \\ x \geq 0 \\ y \geq 0 \end{cases}$

(d) $\begin{cases} x + y \leq 130 \\ x \leq 0.25y \\ x \geq 0 \\ y \geq 0 \end{cases}$

Solution

The answer is (c).

14. Solve the system of equations. If the system has no solution, say that it is inconsistent.

$$\begin{cases} 2x + y = 3 \\ -4x - 2y = -6 \end{cases}$$

(a) $y = -2x + 3$, where x is any real number

(b) $y = 2x + 3$, where x is any real number

(c) $x = -2y + 3$, where y is any real number

(d) The system is inconsistent.

Solution

The equations are equivalent, so any point on the line gives us a solution. Solving either equation for y gives us $y = -2x + 3$.

15. Find the real zeros for the function

$$P(x) = x^3 - x^2 - 17x + 20.$$

(a) 4, $\frac{3 \pm \sqrt{29}}{2}$

(b) 4, $3 \pm \frac{\sqrt{29}}{2}$

(c) 4, $\frac{-3 \pm \sqrt{29}}{2}$

(d) 4, $-3 \pm \frac{\sqrt{29}}{2}$

Solution

If we perform synthetic division with $x - 4$ ($c = 4$) and find a remainder of 0, then we will have $x = 4$ is a zero for $P(x)$.

$$\begin{array}{r|rrrr} 4 & 1 & -1 & -17 & 20 \\ & & 4 & 12 & -20 \\ \hline & 1 & 3 & -5 & 0 \end{array}$$

Now that we know that 4 is a zero for $P(x)$, we can find the other zeros by finding the zeros of the quotient, $x^2 + 3x - 5$. We can find these zeros using the quadratic formula.

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-5)}}{2(1)} = \frac{-3 \pm \sqrt{29}}{2}$$

16. The graph below is the graph of which system in inequalities?

(a) $\begin{cases} y < x^2 + 2x - 3 \\ y > -2x - 3 \end{cases}$

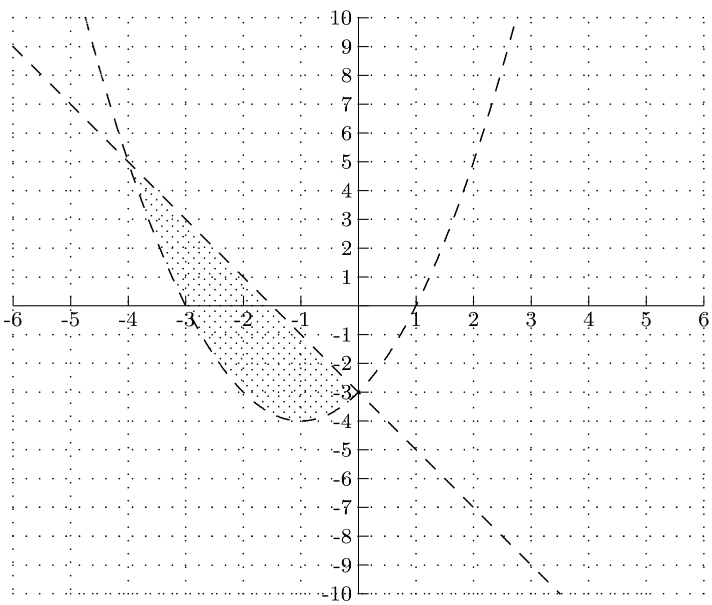
(b) $\begin{cases} y < x^2 + 2x - 3 \\ y < -2x - 3 \end{cases}$

(c) $\begin{cases} y > x^2 + 2x - 3 \\ y > -2x - 3 \end{cases}$

(d) $\begin{cases} y > x^2 + 2x - 3 \\ y < -2x - 3 \end{cases}$

Solution

The shading is above the parabola and below the line, so the graph is the solution to the system in (d).



Free Response

Your work will be graded on this part of the test, so make your work neat and organized. No credit will be given for correct answers without the correct supporting work.

17. \$5000 is deposited into an account earning $7\frac{1}{2}\%$ annual interest, compounded monthly. Find the accumulated amount if the money is left in the account for 6 years. Please give your answer accurate to the nearest dollar. (Recall: $A = P(1 + \frac{r}{n})^{nt}$.)

Solution

$$A = 5000\left(1 + \frac{0.075}{12}\right)^{6(12)} \approx 7831$$

18. Solve the following for x. A decimal approximation will receive no credit.

$$\frac{3x-2}{2x+4} = \frac{5}{8}$$

Solution

Begin by cross-multiplying.

$$\begin{aligned} 8(3x-2) &= 5(2x+4) \\ 24x - 16 &= 10x + 20 \\ 14x &= 36 \\ x &= \frac{36}{14} = \frac{18}{7} \end{aligned}$$

19. The graph of a function f is given below. Give your answers using interval notation. (It is not necessary to show your work on this problem.)

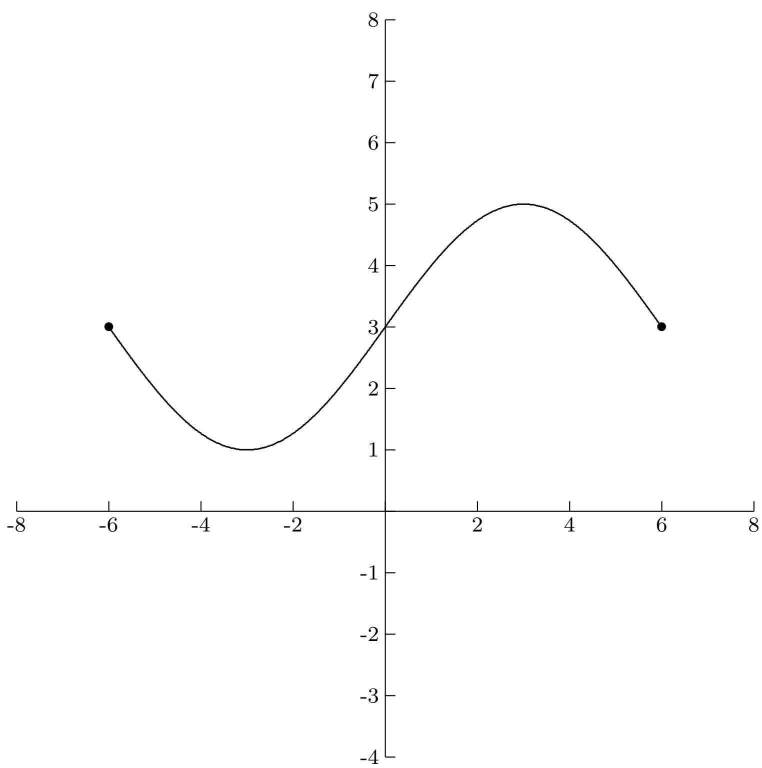
(a) The domain is $\underline{[-6, 6]}$.

(b) The range is $\underline{[1, 5]}$.

(c) Where is this function increasing and where is it decreasing?

i. The function is increasing on the interval(s) $\underline{(-3, 3)}$.

ii. The function is decreasing on the interval(s) $\underline{(-6, -3),\ (3, 6)}$.



20. Consider the rational function

$$R(x) = \frac{x^2 - 4}{x^2 - 9}.$$

(Remember, asymptotes are lines. Give vertical lines and horizontal lines in their proper form to earn full credit for the problems.) It is not necessary to show your work on this problem.

(a) What are the intercepts (both x- and y-intercepts)? (Please give your answers as points, that is, as ordered pairs.)

Solution

We find the x-intercepts by finding the zeros of the numerator, ± 2. We find the y-intercept by evaluating the function at 0: $R(0) = \frac{0^2-4}{0^2-9} = \frac{4}{9}$. The intercepts are $(\pm 2, 0)$ and $(0, \frac{4}{9})$.

(b) What is/are the vertical asymptotes?

Solution

We find the vertical asymptotes by finding the zeros of the denominator, ± 3. The vertical asymptotes are $x = -3$ and $x = 3$.

(c) What is the horizontal asymptote?

Solution

Because the degree of the numerator and denominator are the same, the horizontal asymptote is
$y = \frac{\text{leading coefficient of the numerator}}{\text{leading coefficient of the denominator}}$. The horizontal asymptote is $y = 1$.

21. Determine the point of intersection of the following equations algebraically. You can use your graphing calculator to verify your solutions.

$$\begin{cases} y = x + 2 \\ y = -2x - 1 \end{cases}$$

Solution

We solve this system by substitution.

$$\begin{aligned} x + 2 &= -2x - 1 \\ 3x &= -3 \\ x &= -1 \\ y &= -1 + 2 = 1 \end{aligned}$$

The graphs intersect at the point $(-1, 1)$.

22. Let $f(x) = x^2 - 6x + 8$.
Please show your complete work below.

(a) What is the vertex? ____________

$\boxed{Solution}$

We can find the vertex one of two ways, by completing the square and by using the vertex formula, $h = -\frac{b}{2a}$ and $k = f(h)$.

$$\begin{aligned} f(x) &= x^2 - 6x + 8 \\ f(x) &= (x^2 - 6x + 9) + 8 - 9 \\ f(x) &= (x-3)^2 - 1 \end{aligned}$$

The vertex is $(3, -1)$.

$$h = -\frac{-6}{2(1)} = 3 \quad k = f(h) = 3^2 - 6(3) + 8 = -1$$

(b) What is the y-intercept?____________

$\boxed{Solution}$

The y-intercept is $f(0) = 0^2 - 6(0) + 8 = 8$, or $(0, 8)$.

(c) What are the x-intercepts? ____________

$\boxed{Solution}$

We can find the x-intercepts by factoring (as well as by completing the square and with the quadratic formula).

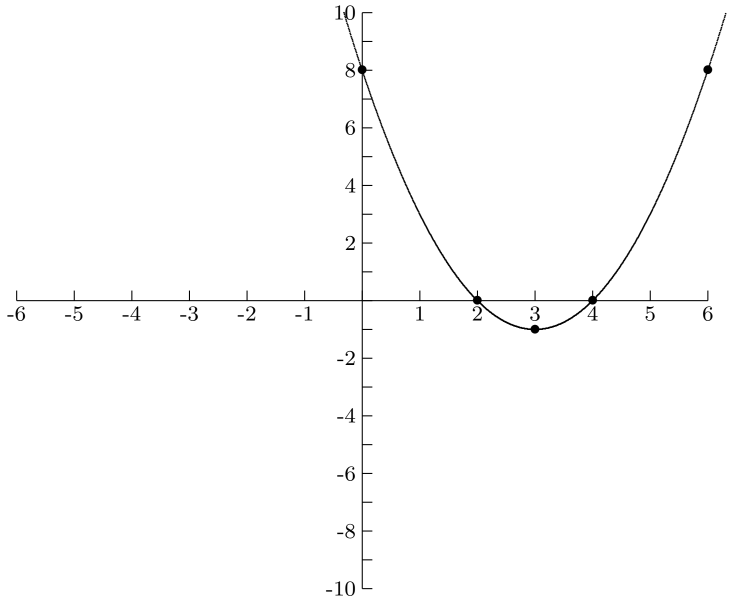
$$f(x) = x^2 - 6x + 8 = (x-4)(x-2)$$

The x-intercepts are $(4, 0)$ and $(2, 0)$.

(d) What is the axis of symmetry (remember, this is a vertical line, so give your answer in the form of a vertical line.)

$\underline{x = 1}$

(e) Use your answers in (a)-(d) to sketch the graph.


10
8
6
4
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-2
-4
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-8
-10
-6 -5 -4 -3 -2 -1 1 2 3 4 5 6