2.61 For a certain style of new automobile, the colors blue, white, black, and green are in equal demand. Three successive orders are placed for automobiles of this style. Find the probabilities of the following events.

a. One blue, one white, and one green are ordered.

b. Two blue are ordered.

c. At least one black is ordered.

d. Exactly two of the orders are for the same color.

Soln.

There are $4^3 = 64$ different possibilities for the colors of the three orders, that's because the first color can be choosen in 4 different ways, the second color can be choosen in 4 different ways, and the third color can be choosen in 4 different ways

$(4)(4)(4) = 4^3$.

a. There are $3!$ ways to arranse the three colors blue, white and green, that is,

the first color can be choosen in 3 different ways,
the second color can be choosen in 2 different ways, an
the third color can be choosen in 1 way.

Therefore $P(\text{one blue one white and one green}) = \frac{3!}{4^3} = \frac{6}{64}$

b. Two blues are ordered.

There are $\binom{3}{2} = 3$ ways to choose the two "blue" orders out of a total of 3 orders, that is:

(blue, blue, color), (blue, color, blue) and (color, blue, blue)

and there are $\binom{3}{1} = 3$ different ways to choose the next color (green, white or black), hence

$$P(\text{two blues}) = \frac{\binom{3}{2}\binom{3}{1}}{4^3} = \frac{3 \cdot 3}{64} = \frac{9}{64}.$$

c. $P(\text{at least one black is demanded}) = 1 - P(\text{no black is demanded}) = 1 - \frac{(3)(3)(3)}{64} = 1 - \frac{27}{64} = \frac{37}{64}.$

d. Two colors must be choosen; one is repeated, the other not. So order matters! There are

first color $\rightarrow$ $4 \times 3 = 12$ ways to do this. For each, there are
second color

$\binom{3}{2} = 3$ ways to choose the two orders that have the same color, thus

$$P(\text{exactly two same color}) = \frac{(12)(3)}{64} = \frac{9}{16}.$$