Test II

Math 3680.002
Nov. 4, 2009

Name: ______________________
by writing my name i swear by the honor code

Read all of the following information before starting the test:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle your final answers for the multiple choice questions.
- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.
- This test has 6 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- Good luck!

1. (*25 points*) The probability mass function for the random variable X is given by the following table:

x	$f_X(x)$	$xf_X(x)$
0	1/10	0
1	1/5	1/5
2	3/10	3/5
3	2/5	6/5

(a) Find the cumulative distribution function for X

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{10} & \text{if } 0 \leq x < 1 \\ \frac{3}{10} & \text{if } 1 \leq x < 2 \\ \frac{6}{10} & \text{if } 2 \leq x < 3 \\ 1 & \text{if } 3 \leq x \end{cases}$$

(b) Fill in the third column of the above table and use them to compute the expected value of X:

$E(X) = 0 + \frac{1}{5} + \frac{3}{5} + \frac{6}{5} = 2$

2. (*15 points*) In a small town, out of 6 accidents that occurs in June 1986, three happened on Friday the 13th. Is this a good reason for a superstitious person to argue that Friday the 13th is inauspicious ?

Let X be # of accidents that would happen on Friday the 13th under the hypothesis H_0 that there is nothing special about Friday the 13th.

$$P(X \geq 3|H_0) = 1 - \sum_{x=0}^{2} \binom{6}{x} (\frac{1}{30})^x (1 - \frac{1}{30})^{6-x} = 1 - 0.9993 = 0.0007$$

Since this P-value is quite small, this is a good reason for a superstitious person to argue that Friday the 13th is inauspicious.

3. (*15 points*) Let X be the number of dots showing up when rolling a fair die once. Find $Var(-X + 10)$.

Since

$$\begin{aligned} E(X) &= 1 \times \frac{1}{6} + \cdots + 6 \times \frac{1}{6} = \frac{7}{2} \\ E(X^2) &= 1^2 \times \frac{1}{6} + 2^2 \times \frac{1}{6} + \cdots + 6^2 \times \frac{1}{6} = \frac{91}{6}, \end{aligned}$$

$$Var(-X + 10) = Var(X) = E(X^2) - (E(X))^2 = \frac{91}{6} - (\frac{7}{2})^2 = \frac{35}{12}.$$

4. (*24 points*) A coin has been bent in such a way that it comes up heads 60% of the time. This bent coin is placed in a box with nine fair coins. A coin is selected randomly from the box. To test if the selected coin is a fair coin, we toss the coin five times and reject the hypothesis of a fair coin if the number of heads coming up is at least four times. In other words, if X denote the number of heads coming up in the five tosses, our decision rule is to reject the fair-coin hypothesis if $X \geq 4$ and retain the fair-coin hypothesis otherwise.

(a) What are the null hypothesis and alternative hypothesis for this problem ?

$H_0 : p = \frac{1}{2}$

$H_a : p > \frac{1}{2}$

(b) Use our decision rule, find α, the probability of a Type I error:

$$\alpha = P(X \geq 4 | H_0) = \sum_{x=4}^{5} \binom{5}{x} (\frac{1}{2})^x (1 - \frac{1}{2})^{5-x} = \frac{3}{16}.$$

(c) Use our decision rule, find β, the probability of a Type II error:

$$\beta = P(X < 4 | H_a) = 1 - P(X \geq 4 | H_a) = 1 - \sum_{x=4}^{5} \binom{5}{x} 0.6^x (1 - 0.6)^{5-x} = 1 - \frac{33696}{100000} = \frac{2072}{3125}.$$

5. (*15 points*) A machine makes faulty widgets 1% of the time. Let X denote the number of faulty widgets in a box of 1,000 widgets. Compute the expected value and variance of X.

$E(X) = np = 1000 \times 0.01 = 10$

$Var(X) = np(1-p) = 1000 \times 0.01 \times (1 - 0.01) = 9.9$

6. *(6 points)*
A man claims to have extrasensory perception (ESP). As a test, a fair coin is flipped 10 times, and the man is asked to predict the outcome in advance. He gets 8 out of 10 correct. Compute the P-value that he would have done <u>at least</u> this well if he had no ESP ?

(a) $\frac{1}{1024}$ (b) $\frac{11}{1024}$ (c) $\frac{7}{128}$ (d) $\frac{45}{1024}$ (e) None of the aforementioned answers.

The correct answer is (c).

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