

Algebra Qualifying Exam Syllabus and Suggested References

Updated 2003

Questions on the Algebra Qualifying Exam will be based on the topics listed in this syllabus. The course sequence Math 5520-5530 will generally cover many, but not all, of the topics listed below.

The bibliographic references at the end of each subject refer to the list of suggested references at the end of the document.

Groups

1. Elementary group actions: groups acting on sets and algebraic objects (groups, rings, vector spaces, etc.), the orbit equation and applications, transitive actions.
2. Isomorphisms, automorphisms, groups of automorphisms, the isomorphism theorem, the correspondence (or lattice isomorphism) theorem.
3. The general structure of groups: the theorems of Lagrange, Cauchy, Cayley, and Sylow, direct and semi-direct products, normal series, solvability, nilpotence, the fundamental theorem of finite abelian groups.
4. Elementary facts concerning p -groups and their structure, the symmetric and alternating groups, the dihedral groups, other groups of rigid motion, matrix groups, free abelian groups.

References for Groups

- Chapter 2 in [1].
- Chapters I and II in [2].
- Chapters 1-8 in [3].
- Chapter 1 in [4].

Rings and Fields

1. Special classes of rings: matrix rings, polynomial rings, Noetherian rings, Artinian rings, local rings.
2. Important substructures: subrings, one-sided and two-sided ideals, prime and maximal ideals, the nilradical, the Jacobson radical.
3. The Chinese Remainder Theorem.
4. Ring isomorphism theorems.
5. Factorizations: UFDs, PIDs, Euclidean domains.
6. Localization and fields of quotients.
7. Polynomial rings: Gauss's lemma, primitive elements, Eisenstein's criterion.
8. Basic facts about fields and field extensions: algebraic, transcendental, normal, separable, and Galois extensions.
9. Splitting fields of polynomials and elementary Galois theory.

References for Rings and Fields

- Chapters 3 and 5 and Section 7.1 in [1].
- Chapter III, Sections VIII.1, VIII.4, IX.2, IX.3, and V.1-6 in [2].
- Chapters 12-14, 16-19, 21, 22, 26, 27 in [3].
- Chapter 2, Sections 4.1-5 and 4.13 in [4].

Modules and Linear Algebra

1. Submodules, quotient modules.
2. Semisimplicity.
3. Linear independence, generating sets, and bases of free modules.
4. Homomorphisms, endomorphisms, isomorphisms, and the isomorphism theorems.

5. Basic linear algebra: the algebra of linear transformations, inner product spaces, orthogonality, Gramm-Schmidt orthogonalization.
6. The algebra of a single linear transformation: matrix representations of linear transformations, characteristic and minimal polynomials, the Cayley-Hamilton theorem, Jordan canonical form.

References for Modules and Linear Algebra

- Chapters 4 and 6 in [1].
- Any undergraduate linear algebra book that includes chapters on inner product spaces.
- Chapter VII and Sections IV.1, IV.2, IV.4, and IV.6 in [2].
- Chapters 11-14 in [3].
- Chapter 3 and Sections 6.1, 6.3, and 6.4 in [4].

References

- [1] I.N. Herstein, *Topics in Algebra*, 2nd ed., Wiley, New York, 1975.
- [2] Thomas Hungerford, *Algebra*, Graduate Texts in Mathematics, vol. 73, Springer-Verlag, New York-Berlin, 1980.
- [3] I.M. Isaacs, *Algebra: A Graduate Course*, Brooks/Cole, Pacific Grove, California, 1994.
- [4] N. Jacobson, *Basic Algebra I*, Second Ed., W.H. Freeman, New York, 1989.