Algebra Qualifying Exam—Syllabus and Suggested References

Questions on the Algebra Qualifying Exam will be based on the topics listed in this syllabus. The course sequence Math 5520–5530 will cover many, but not all, of these topics. The references for each subject area refer to the bibliography at the end. This syllabus will apply to Algebra Qualifying Exams after May 2024.

Groups.

1. Elementary group actions on sets, and on algebraic objects such as groups, rings, vector spaces, etc., the orbit equation and applications, transitive actions
2. Isomorphisms, automorphisms, groups of automorphisms, the isomorphism theorem, the correspondence (or lattice) isomorphism theorem
3. The general structure of groups: the theorems of Lagrange, Cauchy, Cayley, and Sylow, direct and semi-direct products, normal series, solvability, nilpotence, the fundamental theorem of finite abelian groups
4. Elementary facts concerning p-groups and their structure, the symmetric and alternating groups, the dihedral groups, other groups of rigid motions, matrix groups, free abelian groups

References for Groups.

- Chapters 1–6 [DmFt04]
- Chapter 2 in [Her75]
- Chapters I–II in [Hun80]
- Chapters 1–8 in [Isa94]
- Chapter 1 in [Jac89]

Rings and Fields.

1. Special classes of rings: matrix rings, polynomial rings, Noetherian rings, Artinian rings, local rings
2. Important substructures: subrings, one-sided and two-sided ideals, prime and maximal ideals, the nilradical, the Jacobson radical
3. The Chinese remainder Theorem
4. Ring isomorphism theorems
5. Factorization: UFDs, PIDs, Euclidean domains
6. Localization and fields of quotients
7. Polynomial rings: Gauss's lemma, primitive elements, Eisenstein's criterion
8. Basic facts about fields and field extensions: algebraic, transcendental, normal, separable, and Galois extensions
9. Splitting fields of polynomials and elementary Galois

References for Rings and Fields.

- Chapters 7–9, 13, 14, 15.1, 16.1 in [DmFt04]
- Chapters 3, 5, 7.1 in [Her75]
- Chapters III, VIII.1, VIII.4, IX.2, IX.3, V.1–6 in [Hun80]
- Chapters 12–14, 16–19, 21, 22, 26, 27 in [Isa94]
- Chapters 2, 4.1–5, 4.13 in [Jac89]

Modules and Linear Algebra.

1. Submodules, quotient modules, tensor products
2. Semisimplicity
3. Linear independence, generating sets and bases of free modules
4. Homomorphisms, endomorphisms, isomorphisms and the isomorphism theorems
5. Basic linear algebra: the algebra of linear transformations, inner product spaces, orthogonality, Gram-Schmidt orthogonalization
7. The algebra of a single linear transformation: matrix representations of linear transformations, characteristic and minimal polynomials, the Cayley-Hamilton theorem, Jordan canonical form

References for Modules and Linear Algebra.

- Chapters 10–12 in [DmFt04]
- Chapters 4, 6 in [Her75]
- Chapters IV.1, IV.2, IV.4, IV.6, VII in [Hun80]
- Chapters 11–14 in [Isa94]
- Chapters 3, 6.1, 6.3, 6.4 in [Jac89]
- Any linear algebra text covering inner product spaces

References

[DmFt04] D. Dummit and R. Foote, Abstract Algebra, 3rd ed., Wiley, 2004.
[Her75] I. N. Herstein, Topics in Algebra, 2nd ed., Wiley, 1975.
[Hun80] T. Hungerford, Algebra, Springer-Verlag, 1980.
[Isa94] I. M. Isaacs, Algebra: A Graduate Course, Brooks/Cole 1994.
[Jac89] N. Jacobson, Basic Algebra I, 2nd ed., W. H. Freeman, 1989.

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