Millican Colloquium Talk

Title: **The ideal soccer ball**
Speaker: Prof. Peter Doyle (Dartmouth College)
When: Monday 4pm, March 28, GAB 317

Abstract: (Work joint with Scott Crass)

In the course of looking for new ways to solve the quintic using iterated rational maps, we are led to wonder, what is the Platonically ideal shape of a soccer ball?

The familiar 'association football ball' ('soccer ball', for short) is a round relative of a truncated icosahedron. Sanding the vertices off an icosahedron converts its 12 vertices into regular pentagons, and its 20 triangular faces into hexagons. By sanding just the right amount, the hexagons can be made regular. But is this the ideal choice? A truncated icosahedron is not a Platonic solid, whose ideal shape is ordained by its symmetries. Perhaps we should truncate so as to minimize the disparity between the circumscribed and inscribed spheres, or maximize the volume of the associated ideal hyperbolic polyhedron?

As for the ideal soccer ball, instead of simply inflating a truncated icosahedron, perhaps we should look for something more naturally round. In this spirit, we offer as a candidate the picture that emerges from iterating the rational map

$$z \mapsto \frac{(-z^{11} - 66z^6 + 11z)\, H_{20} - wzT_{30}}{(11z^{10} + 66z^5 - 1)\, H_{20} - wT_{30}},$$

where

$$H_{20} = -z^{20} + 228z^{15} - 494z^{10} - 228z^5 - 1,$$

$$T_{30} = z^{30} + 522z^{25} - 10005z^{20} - 10005z^{10} - 522z^5 + 1,$$

and the constant w is one of roots of the polynomial

$$\begin{aligned}
&512578125\, w^{12} - 3865218750\, w^{11} + 23152938750\, w^{10} \\
&-74112921000\, w^9 + 108824537925\, w^8 - 149785512090\, w^7 \\
&+240751261832\, w^6 - 293002530840\, w^5 + 188253684000\, w^4 \\
&+132994820000\, w^3 - 311513400000\, w^2 + 139581600000\, w \\
&-5632000000.
\end{aligned}$$