Solution of the September 2020 Problem of the Month

Correct Solutions were received from :

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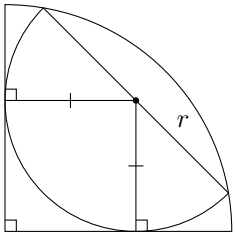
David Duhon's Solution

Problem of the Month, September 2020

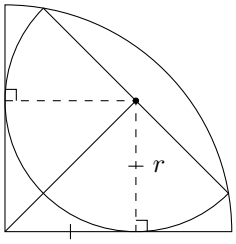
Answer. $R = r\sqrt{3}$.

Proof. We begin by drawing radii from the center of the semicircle to its points of tangency.



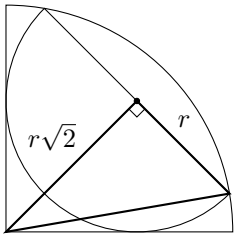
This forms a square with side length r since a circle's tangents are perpendicular to their corresponding radii.

Next we draw a segment connecting the center of the semicircle to that of the quarter circle.



Our new segment is the diagonal of a square with side length r and thus has length $r\sqrt{2}$. We also see, by symmetry, that it forms a right angle with the diameter of the semicircle.

We may then draw a new segment to form the following right triangle:



Applying the Pythagorean theorem, we calcuate the length of its hypotenuse to be $r\sqrt{3}$. But this hypotenuse is a radius of the quarter circle! Thus we have $R = r\sqrt{3}$. □