Problem of the Month, September 2020

**Answer.** \( R = r \sqrt{3} \).

**Proof.** We begin by drawing radii from the center of the semicircle to its points of tangency.

This forms a square with side length \( r \) since a circle’s tangents are perpendicular to their corresponding radii.

Next we draw a segment connecting the center of the semicircle to that of the quarter circle.

Our new segment is the diagonal of a square with side length \( r \) and thus has length \( r \sqrt{2} \). We also see, by symmetry, that it forms a right angle with the diameter of the semicircle.

We may then draw a new segment to form the following right triangle:

Applying the Pythagorean theorem, we calculate the length of its hypotenuse to be \( r \sqrt{3} \). But this hypotenuse is a radius of the quarter circle! Thus we have \( R = r \sqrt{3} \). □