

Weak Inverse Limits

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Abstract

In this talk we define the concept of weak inverse limit and establish some of its elementary properties. Rather than developing the concept and establishing those properties under the most general conditions, we restrict ourselves to weak inverse limits of topological groups.

Theorem 1. *If $\{H_i, A_i; \pi_i^{i+1}\}$ is a weak inverse sequence, the weak inverse limit $\lim_{\leftarrow w} H_i$ is a Polish group.*

We will define a particular kind of weak inverse limit $\lim_{\leftarrow w} H_i^*$. We then can characterize the isometry groups of countable Heine-Borel Polish ultrametric spaces.

Theorem 2. *The isometry group of every countable Heine-Borel Polish ultrametric space is topologically isomorphic to a weak inverse limit $\lim_{\leftarrow w} H_i^*$. Conversely, for every weak inverse limit $\lim_{\leftarrow w} H_i^*$ there exists a countable Heine-Borel Polish ultrametric space X such that $\text{Iso}(X) \cong \lim_{\leftarrow w} H_i^*$.*