6200–Topics in Ergodic Theory; Fall 2023

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The course will cover basics of topological and measure-preserving dynamical systems and ergodic theory. The following topics are intended be covered.

(A) **Topological dynamical systems on compact metric spaces**

- (A1) Topological conjugacy and topological factors.
- (A2) Periodic points, ω and α limit sets.
- (A3) Minimality, transitivity, topological mixing.
- (A4) Symbol dynamical systems and subshifts of finite type.
- (A5) Rotations on the circle, translations on tori and general compact topological groups.
- (A6) Distance expanding maps and the Gauss map.
- (A7) Expansive maps.

(B) **Measure-preserving dynamical systems**

- (B1) Examples of measure-preserving dynamical systems.
- (B2) Bogulubov–Krylov Theorem (about existence of invariant measures).
- (B3) Poincare's Return Theorem.
- (B3) The concept of ergodicity, Birkhoff's Ergodic Theorem, and its numerous consequences.
- (B4) Ergodicity - examples.
- (B5) Mixing properties.
- (B6) Uniquely ergodic systems.
- (B7) First return (induced) maps; Kac Lemma.

(C) **Entropy of measure-preserving dynamical systems**

- (C1) Shannon entropy of a probability vector; Kchinchine's treatment of information theory
- (C2) Entropy of a partition
- (C3) Kolmogorov–Sinaj entropy of a measure-preserving dynamical system; isomorphism problem.

(C4) Tools to calculate Kolmogorov–Sinaj entropy.

(C5) Entropy of concrete systems.

(D) **Entropy and topological pressure of topological dynamical systems**

(D1) The definition.

(D2) Basic properties.

(D3) Entropy and pressure of examples; relation to spectral radii of matrices.

(D4) Variational Principle.

There will be no single textbook. I will deliver pdf files of 1-2 of my books, whose parts will be used for the course. Some solid familiarity with measure theory and topology of metrizable spaces would be very helpful. The course will count for breadth requirement of topology. Grading will be based on participation and activity in the class.