

Topology Qualifying Exam Syllabus and Suggested Reading

The students taking a topology qualifying exam are assumed to be familiar with definitions, theorems, techniques of proofs, and examples appropriate for the topics listed below.

1. Preliminaries from Set Theory

- Algebra of sets, cardinal numbers, ordinal numbers, axiom of choice, well-ordered sets, transfinite induction, Zorn lemma.

2. Topological Spaces

- Topological spaces, open and closed sets, bases of topology, closures and interiors of sets, derivatives of sets, dense and boundary sets, Borel sets, G_δ and F_σ sets, separation axioms, Urysohn's lemma and Tietze's extension theorem, nets and filters, subspaces, axioms of countability, disjoint unions, Cartesian products, perfect spaces.

3. Maps of Topological Spaces

- Continuous maps and functions, open and closed mappings, homeomorphisms, quotient spaces and mappings.

4. Compact Spaces

- Compact spaces, closed subspaces, Cartesian products, Tychonoff theorem on Cartesian products, locally compact spaces, compactifications (especially one-point [Alexandrov] and Stone-Cech compactifications), continuous images of compact spaces, perfect mappings, countably compact spaces, sequentially compact spaces, Lindelöf spaces.

5. Metric and Metrizable Spaces

- Metric and metrizable spaces, subspaces, countable Cartesian products, compactness, various conditions equivalent to compactness, totally bounded spaces, separable spaces, complete spaces, Baire Category Theorem, metrization theorems.

6. Connected Spaces

- Connected spaces, connected subspaces of the real line, unions of connected spaces, continuous images of connected spaces, continua (compact connected spaces, intersections of continua), arcwise connected spaces, locally connected spaces, totally disconnected spaces, topological characterization of the Middle-Third Cantor set.

Suggested Reading

- N. Bourbaki, *General Topology*
- R. Engelking, *General Topology*
- J. L. Kelley, *General Topology*
- J. Munkres, *Topology, A First Course*
- J. Willard, *Topology*
- J. Dugundji, *Topology*