On the local dimension of the set of univoque bases

Fix a positive integer M. The univoque set $\mathscr{U}$ of bases $q \in (1, M+1)$ in which the number 1 has a unique expansion over the alphabet $\{0, 1, \dots, M\}$ plays a central role in the theory of non-integer base expansions. It has Lebesgue measure zero but Hausdorff dimension one. However, the dimension of $\mathscr{U} \cap I$ varies greatly with the choice of the subinterval I. One way to capture this variation is through the function

$$f(q) := \lim_{\delta \to 0} \dim_H \big(\mathscr{U} \cap (q-\delta, q+\delta)\big), \qquad q \in (1, M+1).$$

In this talk I will give a precise characterization of the function $f(q)$ and its one-sided analogs, and show how at least for certain values of q they can be explicitly calculated. A consequence of the main result is that f is continuous at those (and only those) points where it vanishes. Using the function f we can express the Hausdorff dimension of $\mathscr{U} \cap I$ for any subinterval I. Finally, the main result is applied to solve an open problem about strongly univoque sets.

This is joint work with Derong Kong.