RTG Logic and Dynamics

UNDERGRADUATE MATHEMATICS RESEARCH COLLOQUIUM SERIES

Tuesday, October 2, 2012

5:00-6:00 PM

General Academic Building, Room 104

A pre-lecture reception with cookies, coffee and tea will be held at 4:30 PM in the General Academic Building, Room 472 .

The ***RTG in Logic & Dynamics*** is a research training group supported by the National Science Foundation and the University of North Texas.

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Manfred Denker received his PhD at the University of Erlangen/Germany in 1972 and is currently a professor at The Pennsylvania State University as well as an adjunct professor at Case Western Reserve University. He has authored five books addressing his main mathematical interests: ergodic theory, dynamical systems, nonparametric statistics and probability theory. In addition, he's published more than 100 research articles and his latest interests comprise brain research, gene analysis and financial mathematics. Dr. Denker was awarded the Humboldt prize in 2002 and the Senior Award of the Japanese Society for Promotion of Science in 1996.

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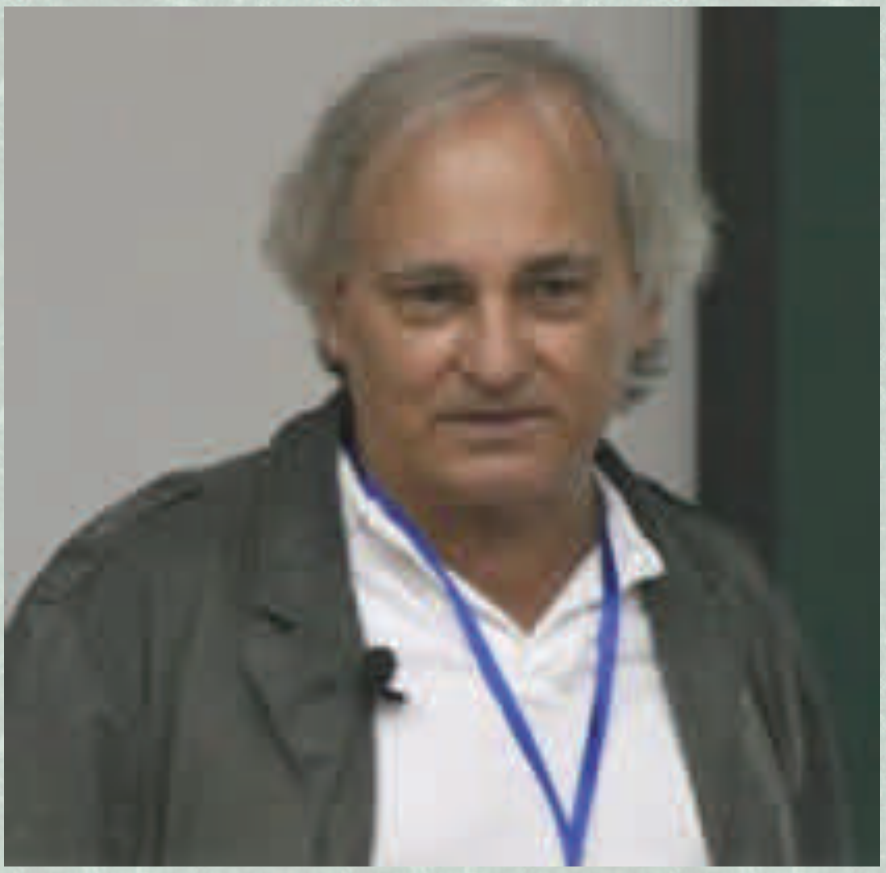

LOCAL LIMIT THEOREMS IN PROBABILITY

First of all, a local limit theorem is actually a topic in Fourier analysis, the talk will, however, restrict to results which are easily accessible with undergraduate mathematics and some understanding of probability.

The whole story begins with Jacob Bernoulli's law of large numbers for a series of coin tossing (published in 1713). It says that the number $k(n)$ of heads in n trials differs from $n/2$ by at most some fixed amount is an event which becomes more and more likely as n increases, and its probability approaches one. De Moivre made this more precise in 1738 by showing that the event that $k(n) = n/2 + d$ is asymptotic to

$$\frac{\sqrt{2}}{\sqrt{n\pi}} \mathrm{e}^{-d^2/n}. \tag{1}$$

This is the classical local limit theorem which will explained in detail. In particular, $k(n)$ has an analytic formulation as it equals$\binom{n}{\frac{n}{2}+d}4^{-n}$.

At the end of the talk a recently found new result will be mentioned that de Moivre's result has an almost sure version in terms of logarithmic averages: The expressions

$$\frac{1}{\ln N} \sum_{n=1}^{N} \frac{1}{2\sqrt{n}} \mathbb{I}_{\{k(n)=n/2+d(n)\}}$$

approach the expression in (1) with d^2/n replaced by x^2 where $d(n)^2/n \asymp x^2$. In particular, since the the logarithmic average is computable from the experimental data the exponential function in (1) can be calculated.