## PROBLEM OF THE MONTH
## PROBLEM FOR OCTOBER 2015

Let $T$ be an equilateral triangle. Let $P$ be an arbitrary point in $T$. Let $d_1$, $d_2$, and $d_3$ be the distances of $P$ to each of the three sides of $T$. Show that $d_1 + d_2 + d_3$ is independent of $P$! That is, show that $d_1 + d_2 + d_3$ is the same regardless of which point $P$ is chosen.

**Solution:**

$$d_1 + d_2 + d_3 = \frac{\sqrt{3}}{2}s$$

where $s$ is the length of a side of the triangle.

There were several correct solutions turned in this month. Nice going everyone! A few of them gave an argument similar to the following.

Let $P$ be an arbitrary point in the equilateral triangle $XYZ$ where $s$ is the length of a side of the triangle. Then it is known that the area of triangle $XYZ$ is $\frac{\sqrt{3}}{4}s^2$. Next we drop a perpendicular of length $d_1$ from $P$ to say side $YZ$. Calculating the area of $PYZ$ we see we get $\frac{1}{2}sd_1$. Similarly we drop a perpendicular of length $d_2$ from $P$ to $XY$. Calculating the area of $PXY$ we obtain $\frac{1}{2}sd_2$. Finally we do the same with $d_3$ and triangle $PXZ$. Its area is $\frac{1}{2}sd_3$. Adding the areas of the three small triangles we see that this is equal to the area of $XYZ$ which is $\frac{\sqrt{3}}{4}s^2$. Thus we obtain: $\frac{\sqrt{3}}{4}s^2 = \frac{1}{2}s(d_1 + d_2 + d_3)$. After rewriting we obtain:

$$d_1 + d_2 + d_3 = \frac{\sqrt{3}}{2}s.$$