PROBLEM OF THE MONTH
PROBLEM FOR OCTOBER 2015

Let \( T \) be an equilateral triangle. Let \( P \) be an arbitrary point in \( T \). Let \( d_1, d_2, \) and \( d_3 \) be the distances of \( P \) to each of the three sides of \( T \). Show that \( d_1 + d_2 + d_3 \) is independent of \( P \)!
That is, show that \( d_1 + d_2 + d_3 \) is the same regardless of which point \( P \) is chosen.

Solution:

\[
d_1 + d_2 + d_3 = \frac{\sqrt{3}}{2} s
\]

where \( s \) is the length of a side of the triangle.

There were several correct solutions turned in this month. Nice going everyone! A few of them gave an argument similar to the following.

Let \( P \) be an arbitrary point in the equilateral triangle \( XYZ \) where \( s \) is the length of a side of the triangle. Then it is known that the area of triangle \( XYZ \) is \( \frac{\sqrt{3}}{4} s^2 \). Next we drop a perpendicular of length \( d_1 \) from \( P \) to say side \( YZ \). Calculating the area of \( PYZ \) we see we get \( \frac{1}{2} s d_1 \). Similarly we drop a perpendicular of length \( d_2 \) from \( P \) to \( XY \). Calculating the area of \( PXY \) we obtain \( \frac{1}{2} s d_2 \). Finally we do the same with \( d_3 \) and triangle \( PXZ \). Its area is \( \frac{1}{2} s d_3 \). Adding the areas of the three small triangles we see that this is equal to the area of \( XYZ \) which is \( \frac{\sqrt{3}}{4} s^2 \). Thus we obtain: \( \frac{\sqrt{3}}{4} s^2 = \frac{1}{2} s (d_1 + d_2 + d_3) \). After rewriting we obtain:

\[
d_1 + d_2 + d_3 = \frac{\sqrt{3}}{2} s.
\]