SOLUTION FOR DECEMBER 2016 PROBLEM

Find all solutions of:
\[3\sqrt{x + 9} - 3\sqrt{x - 9} = 3.\]

**SOLUTION:**
\[x = \pm 4\sqrt{5}.\]

Correct solutions were turned in by: William Liu, Xiangyu Kong, and Ethan Seal.

Cubing both sides gives of the equation gives:
\[x + 9 - 3\sqrt{(x + 9)^2} \sqrt{x - 9} + 3\sqrt{x + 9} \sqrt{(x - 9)^2} - (x - 9) = 27.\]
Thus:
\[-\sqrt{(x + 9)^2} \sqrt{x - 9} + \sqrt{x + 9} \sqrt{(x - 9)^2} = 3.\]
Rewriting this we get:
\[-\sqrt{x + 9} \sqrt{x - 9} \left( \sqrt{x + 9} - \sqrt{x - 9} \right) = 3.\]
Now notice that the term in the parenthesis is 3 (look at the original statement of the problem).
Thus:
\[\sqrt{x + 9} \sqrt{x - 9} = -1.\]
Cubing both sides again gives:
\[x^2 - 81 = -1.\]
Therefore:
\[x^2 = 80 = 16 \cdot 5.\]
Thus:
\[x = \pm 4\sqrt{5}.\]