Solution: Prove that $r = A'E$.

**Proof:** Let $P$ be the point between $A$ and $D$ where paper was folded and $Q$ is the point between $B$ and $C$ where paper was folded. Let $z = AB$, $x = AP$. So $PA' = x$, $A'E = w$, $PE = \sqrt{x^2 + w^2}$, $EB' = z - w$. So we want to show $r = w$.

Next note that the triangles $PA'E$, $B'DE$ and $QCB'$ are all similar. Since the first two are similar then we see $ED = cw$, $DB' = cx$, $EB' = c\sqrt{x^2 + w^2}$ for some $c > 0$.

Next since the length of the segment $AB$ is the same as the length of the segment $AD$ then we have:

$$z = x + \sqrt{x^2 + w^2} + cw.$$ 

Also since the length of the segment $AB$ is the same as the length of the segment $A'B'$ then we have:

$$w + c\sqrt{x^2 + w^2} = z.$$

Equating these and solving for $c$ gives:

$$c = \frac{x}{\sqrt{x^2 + w^2} - w} + 1.$$ 

Multiplying by $\frac{\sqrt{x^2 + w^2} + w}{\sqrt{x^2 + w^2} + w}$ gives:

$$c = \frac{x}{\sqrt{x^2 + w^2} + w} + 1$$

so:

$$cx = \sqrt{x^2 + w^2} + x + w. \quad (1)$$

Now using the fact mentioned in the hint that:

$$r = \frac{2\text{Area}(B'DE)}{\text{Perimeter}(B'DE)}$$

gives:

$$r = \frac{c^2 x w}{c(\sqrt{x^2 + w^2} + x + w)} = \frac{cxw}{\sqrt{x^2 + w^2} + x + w}. \quad (2)$$

Finally by (1)-(2) we get:

$$r = \left(\frac{cx}{\sqrt{x^2 + w^2} + x + w}\right)w = 1 \cdot w = w.$$ 

$\square$