SOLUTION FOR APRIL 2021

Correct solutions were submitted by:

Find a simple formula for:

$$1\binom{n}{1}+2\binom{n}{2}+3\binom{n}{3}+\cdots+n\binom{n}{n}.$$

SOLUTION:

$$n2^{n-1}=1\binom{n}{1}+2\binom{n}{2}+3\binom{n}{3}+\cdots+n\binom{n}{n}.$$

Proof: The binomial theorem states:

$$(x+y)^n=\sum_{k=0}^{n}\binom{n}{k}x^k y^{n-k}.$$

Subsituting $y=1$ gives:

$$(x+1)^n=\sum_{k=0}^{n}\binom{n}{k}x^k.$$

Differentiating we obtain:

$$n(x+1)^{n-1}=\sum_{k=1}^{n}k\binom{n}{k}x^{k-1}.$$

Letting $x=1$ gives:

$$n2^{n-1}=\sum_{k=1}^{n}k\binom{n}{k}.$$