A correct solution was submitted by:

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Let $T$ be a right triangle with sides $a, b$, and hypotenuse $c$. Draw the incircle with radius $r$. Denote the vertices as $L, Q, R$ with vertex $Q$ at the right angle. Let $S$ be the intersection where the incircle meets the hypotenuse. Show that:

$$LS = \frac{1}{2}(c + a - b).$$

HINT: It is known that if $A$ is the area and $P$ the perimeter then $A = \frac{1}{2}rP$.

SOLUTION: Using $A = \frac{1}{2}rP$ gives $\frac{1}{2}ab = A = \frac{1}{2}rP = \frac{1}{2}r(a + b + c)$. Thus $ab = r(a + b + c)$. At the right angle we have a square with side $r$. Let $X$ be the center of the incircle. And denote the square with side $r$ as $QUXV$ where $U$ is on $LQ$ and $V$ is on $QR$. Then $LUX$ is congruent to $LSX$ and also $RVX$ is congruent to $RSX$. This then implies $a - r + b - r = c$ so:

$$a + b - c = 2r. \quad (1)$$

Next we have $LS = a - r$ and so using (1) gives $2LS = 2a - 2r = 2a - (a + b - c) = c + a - b$ and thus:

$$LS = \frac{1}{2}(c + a - b).$$