SOLUTION FOR MARCH 2016

Find all functions which satisfy:

\[ f(x) + 2f\left(\frac{1}{1-x}\right) = x. \]

**SOLUTION:**

\[ f(x) = \frac{1}{9} \left[ x + 4 - \frac{4}{x} - \frac{2}{1-x} \right]. \]

We begin with:

\[ f(x) + 2f\left(\frac{1}{1-x}\right) = x. \] (1)

Replacing \( x \) with \( \frac{1}{1-x} \) gives:

\[ f\left(\frac{1}{1-x}\right) + 2f\left(1 - \frac{1}{x}\right) = \frac{1}{1-x} \] (2)

Now replacing \( x \) with \( 1 - \frac{1}{x} \) gives:

\[ f\left(1 - \frac{1}{x}\right) + 2f(x) = 1 - \frac{1}{x}. \] (3)

Multiplying equation (2) by -2 and adding to equation (1) gives:

\[ f(x) - 4f\left(1 - \frac{1}{x}\right) = x - \frac{2}{1-x}. \] (4)

Multiplying equation (3) by 4 and adding to equation (4) gives:

\[ 9f(x) = x + 4 - \frac{4}{x} - \frac{2}{1-x}. \]

Finally dividing by 9 yields the result.