Let $x \geq 0, y \geq 0, \text{ and } z \geq 0$. Find all solutions of:

\[
\begin{align*}
    x^{1/3} - y^{1/3} - z^{1/3} &= 16 \\
    x^{1/4} - y^{1/4} - z^{1/4} &= 8 \\
    x^{1/6} - y^{1/6} - z^{1/6} &= 4.
\end{align*}
\]

**SOLUTION:** The only solution is:

\[
x = 4096, y = 0, z = 0.
\]

We first introduce new variables: $a = x^{1/12}, b = y^{1/12}, \text{ and } c = z^{1/12}$ so that the above equations become:

\[
\begin{align*}
    a^4 - b^4 - c^4 &= 16 \\
    a^3 - b^3 - c^3 &= 8 \\
    a^2 - b^2 - c^2 &= 4.
\end{align*}
\]

Rewriting the first and third equations we see that:

\[
\begin{align*}
    a^4 &= 16 + b^4 + c^4 \\
    a^2 &= 4 + b^2 + c^2.
\end{align*}
\]

Squaring this second equation and equating it to the first equation gives:

\[
16 + b^4 + c^4 = a^4 = 16 + b^4 + c^4 + 8b^2 + 8c^2 + 4b^2c^2.
\]

Thus we see that:

\[
4b^2 + b^2c^2 + 4c^2 = 0.
\]

Since each of these terms is nonnegative we see that the only solution of this equation is $b = c = 0$. Substituting this into the original equations gives:

\[
\begin{align*}
    a^4 &= 16 \\
    a^3 &= 8 \\
    a^2 &= 4
\end{align*}
\]

and the only nonnegative solution of this is $a = 2$. Returning to the original variables $x, y, \text{ and } z$ we see that the only solution of the original system of equations is:

\[
x = 2^{12} = 4096, y = z = 0.
\]