Logic, Dynamics and Their Interactions, with a Celebration of the Work of Dan Mauldin

University of North Texas
June 4–8, 2012

All lectures will be held in 155 Business Leadership Building (BLB). The reception will be held on Monday, June 4, in the Golden Eagle Suite of the University Union. The banquet will take place on Wednesday, June 6, in Silver Eagle A of the University Union.

Monday, June 4

8:15–9:15: Registration
9:15–9:30: Opening remarks
9:30–10:30: Special address: **John Steel** (Berkeley), *The hereditarily ordinal definable sets in models of determinacy.*
10:45–11:45: Minicourse lecture: **James Yorke** (College Park), *The many aspects of chaos.*
11:45–2:00: Lunch break
2:00–2:35: Special session lecture: **Aaron Hill** (UNT), *Topological isomorphism for non-degenerate rank-1 systems.*
2:45–3:20: Special session lecture: **Brandon Seward** (Michigan), *Group colorings and Bernoulli subflows.*
3:30–4:05: Special session lecture: **Jack Lutz** (ISU), *The Dimensions of Individual Points in Euclidean Space.*
4:05–4:30: Coffee break
4:30–5:30: Plenary lecture: **Alexander Kechris** (Caltech), *Dynamics of non-Archimedean Polish groups.*
5:40–6:15: Special session lecture: **Todor Tsankov** (Université Paris Diderot), *The minimal flows of* S_∞.
6:30–8:00: Reception

Tuesday, June 5

8:15–8:45: Coffee
8:45–9:45: Minicourse lecture: **James Yorke** (College Park), *Three theorems with similar proofs.*
10:00–11:00: Plenary lecture: **Vitaly Bergelson** (OSU), *Uniform distribution, generalized polynomials and translations on nil-manifolds.*

The conference is partially supported by the National Science Foundation grant DMS-0943870 and the University of North Texas.

11:15–12:15: Plenary lecture: **Kenneth Falconer** (St. Andrews), *A selective survey of self-similar sets and their relatives.*

12:15–2:00: Lunch break

2:00–2:35: Special session lecture: **Lars Olsen** (St. Andrews), *Multifractal Analysis. A brief survey.*

2:45–3:20: Special session lecture: **Ka-Sing Lau** (CUHK), *Iterated function systems and tree structure.*

3:30–4:05: Special session lecture: **Arnold Miller** (Madison), *Universal functions.*

4:05–4:30: Coffee break

4:30–5:30: Special address: **Steve Jackson/Mariusz Urbański** (UNT), *A report on selected research activities of Dan Mauldin.*

5:30–6:30: Forum

Wednesday, June 6

8:15–8:45: Coffee

8:45–9:45: Minicourse lecture: **James Yorke** (College Park), *Plato's allegory of the cave.*

10:00–11:00: Plenary lecture: **Sławomir Solecki** (UIUC), *Point realizations of Boolean actions.*

11:15–12:15: Minicourse lecture: **Asger Törnquist** (Copenhagen), *The conjugacy problem in ergodic theory: a descriptive set-theoretic approach. Part 1.*

12:15–1:30: Lunch break

1:30–6:00: Excursion

7:00–10:00: Banquet

Thursday, June 7

8:15–8:45: Coffee

8:45–9:45: Minicourse lecture: **Asger Törnquist** (Copenhagen), *The conjugacy problem in ergodic theory: a descriptive set-theoretic approach. Part 2.*

10:00–11:00: Plenary lecture: **Doug Hensley** (TAMU), *Computing the vital statistics of a class of continued fraction dynamical systems.*

11:15–12:15: Plenary lecture: **Lewis Bowen** (TAMU), *Sofic entropy theory: a review with updates.*

12:15–2:00: Lunch break

2:00–2:35: Special session lecture: **Siegfried Graf** (Universität Passau), *Local properties of optimal quantization for absolutely continuous probabilities.*

2:45–3:20: Special session lecture: **Stanley Williams** (USU), *Guass-Green theorems for open sets with fractal boundaries generated by graph-directed constructions.*

3:30–4:05: Special session lecture: **Zoltán Buczolich** (Eötvös Loránd University), *Equi-kneading of skew tent maps in the square.*

4:05–4:30: Coffee break
4:30–4:45: Contributed talk: **Pieter Allaart** (UNT), *The Hausdorff dimension of level sets of generalized Takagi functions.*
4:50–5:05: Contributed talk: **Karl Backs** (UNT), *Disintegration of σ-finite measures.*
5:10–5:25: Contributed talk: **James Kuodo Huang** (Association of International Uncertainty Computing), *Hilbert logic versus Boolean logic.*
5:30–5:45: Contributed talk: **Kiko Kawamura** (UNT), *The Takagi function revisited.*

Friday, June 8

8:15–8:45: Coffee
8:45–9:45: Minicourse lecture: **Asger Törnquist** (Copenhagen), *The conjugacy problem in ergodic theory: a descriptive set-theoretic approach. Part 3.*
10:00–11:00: Plenary lecture: **Randall Dougherty** (CCR), *Translating the Cantor set by a random.*
11:15–11:50: Special session lecture: **Andrew Yingst** (SC), *Homeomorphic and good Bernoulli trial measures.*
11:50–1:30: Lunch break
1:30–1:45: Contributed talk: **Mario Roy** (York University), *Conformal graph directed Markov systems: Recent advances.*
1:50–2:05: Contributed talk: **Mrinal Roychowdhury** (UTPA), *Hausdorff and upper box dimension estimate of hyperbolic recurrent sets.*
2:10–3:10: Plenary lecture: **Benjamin Weiss** (Hebrew University of Jerusalem), *The isomorphism problem for non-free actions.*
3:10–3:25: Closing Remarks

Abstracts of Plenary Talks

Vitaly Bergelson, *Uniform distribution, generalized polynomials and translations on nil-manifolds.* Ohio State University. `vitaly@math.ohio-state.edu`.

A classical theorem due to H. Weyl states that if P is a polynomial over R such that at least one of its coefficients (other than the constant term) is irrational, then the sequence $P(n), n = 1, 2, \ldots$ is uniformly distributed mod 1. We will discuss some recent extensions of this theorem which involve "generalized polynomials", that is, functions which are obtained from the conventional polynomials by the use of the greatest integer function, addition and multiplication. We will explain the role of dynamical systems on nil-manifolds in obtaining these results and discuss some of the connections with and applications to combinatorics and number theory.

Lewis Bowen, *Sofic entropy theory: a review with updates.* Texas A&M University. lpbowen@math.tamu.edu.

Sofic entropy is a generalization of the classical Kolmogorov-Sinai entropy to actions of sofic groups, which is a large class of groups containing many non-amenable groups. It helps give a complete classification of Bernoulli shifts up to measure-conjugacy for a large class of groups. However, it does not satisfy many of the nice properties of Kolmogorov-Sinai entropy, such as additivity under direct products, in general. This leads to the problem of obtaining such formulae under restrictions on the sofic approximation (and the group).

Randall Dougherty, *Translating the Cantor set by a random.* Center for Communications Research–La Jolla.
rdough@ccrwest.org.

This talk will describe the concept of constructive dimension of real numbers (and its relation to Hausdorff dimension, randomness, and Kolmogorov complexity); then it will describe how much randomness can be cancelled from a real by adding to it a member of the Cantor set (or, equivalently, what are the possible constructive dimensions of points in a translate of the Cantor set). The methods used combine Hausdorff dimension and Kolmogorov complexity techniques with a combinatorial construction of low-dimension additive complements of the Cantor set. The methods allow one to find the Hausdorff dimension of the set of points in a Cantor set translate with a given constructive dimension.

Kenneth Falconer, *A selective survey of self-similar sets and their relatives.* University of St. Andrews. kjf@st-andrews.ac.uk.

Self-similar fractals have been around for many years, but since the 1980s they have been studied intensively, particular from the viewpoints of their dimensions and geometrical structure. Many variants have been introduced, such as statistically self-similar fractals, self-conformal sets, self-affine sets, graph-directed variants, multifractal analogues, etc, etc. We will give a necessarily selective survey of this now vast area to which Dan Mauldin has made major contributions.

Doug Hensley, *Computing the vital statistics of a class of continued fraction dynamical systems.* Texas A&M University. dhensley@math.tamu.edu.

The standard continued fraction map T taking x to $1/x - \lfloor 1/x \rfloor$ is associated with a dynamical system that has an invariant density (the Gauss density), an entropy $(\pi^2/(12\log 2))$, a Lyapunov or decay constant, and behind all those, a transfer operator L that replaces an input density function for the distribution of X on $[0,1]$ with an output density function for the density of TX.

We consider variants in which one has a subset D of $[0,1]$ and a map T_D that takes x, as before, to $y = 1/x - \lfloor 1/x \rfloor$ if $y \notin D$, but to $1-y$ if $y \in D$. Typically, one still has an invariant density and so forth, but closed form descriptions of these are generally unavailable. So, can we compute them?

Here, we give an approach that yields these things to 20 or 50 digits or more, provided that D or a related set D' consists of a finite number of intervals with rational endpoints, and that 0 is in the interior of the complement of D. The basic idea is to cut $[0,1]$ into a finite number of intervals, think of a (suitably nice) density function on $[0,1]$ as an ensemble of analytic functions, one per interval, and think of the transfer operator as an infinite matrix tied to the coefficients in the series expansions of the functions in the ensemble. It turns out that the matrix can safely be pruned to tractably finite size while preserving many digits accuracy.

The main drawbacks are that for complicated sets D, many intervals will be needed, and that rational endpoints seem to be essential.

This work grew out of joint work with K. Dajani and C. Kraaikamp, carried out in part during a visit to T.U. Delft, Netherlands, in 2011.

Alexander Kechris, *Dynamics of non-Archimedean Polish groups.* California Institute of Technology. kechris@caltech.edu.

In recent years there has been considerable activity in the study of the dynamics of Polish non-archimedean groups and this has led to interesting interactions between logic, finite combinatorics, group theory, topological dynamics, ergodic theory and representation theory. In this talk I will give a survey of some of the main directions in this area of research.

Sławomir Solecki, *Point realizations of Boolean actions.* University of Illinois at Urbana-Champaign. ssolecki@math.uiuc.edu.

We will look at measure preserving Boolean actions of Polish groups and consider the problem, going back to Mackey, of determining when such actions have point realizations. We will explore the boundary line between the groups whose all Boolean actions have point realizability and those that do not have this property. One result, joint with Kwiatkowska, states that Boolean action of Polish groups of isometries of locally compact separable metric spaces can always be point realized. On the other hand, a very recent result with Moore, states that the group of all continuous functions from an uncountable compact space to the circle does not have the point realizability property. In several respects, this last group is quite different from other groups that were shown earlier, by Vershik, Becker and Glasner-Weiss, not to have the point realizability property. Connections with the solution to Hilbert's 5-th problem, with the concentration of measure phenomena, and with the Cameron-Martin theorem will be mentioned.

Benjamin Weiss, *The isomorphism problem for non free actions.* Hebrew University of Jerusalem. weiss@math.huji.ac.il.

For a countable group G the space of subgroups has a natural topology which makes it compact, and G acts on this compact space by conjugation. Any action by G on a space X will define a factor action on the subgroups by mapping points in X to their stabilizers. In particular any measure preserving action of G will give rise in this way to a conjugation invariant measure on the space of subgroups. Such measures have received a lot of attention recently under the name Invariant Random Subgroups (IRS). If the action is free then this measure concentrates on the identity. The isomorphism problem for non free actions is the classical isomorphism problem restricted to those non free actions associated with a fixed non trivial IRS. I will survey what little I know about this problem, including an analogue of Ornstein's theorem for nonfree actions of arbitrary countable groups.

Abstracts of Minicourses

Asger Törnquist, *The conjugacy problem in ergodic theory: a descriptive set-theoretic approach.* University of Copenhagen. asgert@math.ku.dk.

In recent years there has been a large amount of interest in the descriptive set theory of the conjugacy relation for measure preserving, ergodic group actions. At first, the focus was on the classification problem, where descriptive set theoretic techniques were used by authors like Foreman, Hjorth and Weiss to rule out a reasonable classification by certain kinds of invariants. However, spurred by a recent paper of Foreman, Rudolph and Weiss, the focus has now shifted towards the decision problem of deciding when two ergodic, measure preserving actions of a group are conjugate. This problem can fruitfully be approached using the distinction between Borel and analytic sets in descriptive set theory.

In these talks, I will give an overview of these developments. One of the goals of the talks will be to discuss a recent result by Epstein and Tornquist, who have shown that for countable groups containing a free non-amenable group normally, the conjugacy relation on ergodic, probability measure preserving actions is complete analytic. I will also discuss the decision problem for some other important equivalence relations in ergodic theory, namely orbit equivalence and von Neumann equivalence. The Epstein-Tornquist result extends to these (coarser) equivalence relations as well, but interestingly, the proof of this relies on some serious set theory in the form of forcing absoluteness.

James Yorke, *The many aspects of chaos*; *Three theorems with similar proofs*; *Plato's allegory of the cave.* University of Maryland. yorke@math.umd.edu.

The many aspects of chaos: This is an introduction to the various ways in which chaos manifests itself.

Three theorems with similar proofs: I will discuss three theorems that have similar proofs, beginning with the Brouwer Fixed Point theorem and ending with period-doubling cascades in dynamical systems.

Plato's allegory of the cave: People sometimes divide science into two topics: theory and experiment. Plato emphasizes that our observational powers are quite limited. This talk is on the theory of experiments, addressing what can be learned from observation (as a topic in mathematics).

Abstract of Special Address

John Steel, *The hereditarily ordinal definable sets in models of determinacy.* University of California, Berkeley. steel@math.berkeley.edu.

If M is a model of the Axiom of Determinacy, then universe HOD^M of all sets hereditarily ordinal definable in M is of great interest. HOD^M is a model of ZFC plus "there are Woodin cardinals" which, roughly speaking, has the same information as M.

If the determinacy model M is not too large, then its HOD admits a fine-structural analysis parallel to fine-structural analysis of canonical extender models with large cardinals. (For example, HOD satisfies the GCH.) There are, however, some important new features. How to extend the fine-structural analysis of HOD to larger determinacy models is one of the central open problems in pure large cardinal theory.

In this lecture, we shall give an overview of the fine structure of HOD^M in the region where we know it, and try to explain why it is important to extend this region further.

Abstracts of Special Sessions Talks

Zoltán Buczolich, *Equi-kneading of skew tent maps in the square.* Eötvös Loránd University. `buczo@cs.elte.hu`.

Skew tent maps $T_{\alpha,\beta} : [0,1] \to [0,1]$ are considered with vertex at $(\alpha,\beta) \in [0,1]^2$. Let $U \subset [0,1]^2$ be the open triangle bounded by $y = x$, $y = 1 - x$ and $y = 1$.

It is well-known that the topological entropy $h(\alpha,\beta)$ of $T(\alpha,\beta)$ is monotone increasing if α is fixed and β increases, as long as $(\alpha,\beta) \in U$. The starting point of our talk is the question about the behavior of the topological entropy if we fix β and let α change. Parameters $(\alpha,\beta) \in U$ corresponding to equal topological entropy also correspond to equal kneading sequence. That is, we need to study properties of "equi-kneading curves" in the parameter space. With a different parametrization these curves have already been studied by M. Misiurewicz and E. Visinescu. We explore the relationship between the different parametrizations and discuss some new results.

This is joint work with Gabriella Keszthelyi.

Siegfried Graf, *Local properties of optimal quantization for absolutely continuous probabilities.* Universität Passau. `graf@fim.uni-passau.de`.

This talk reports on joint work with H. Luschgy, Trier and G. Pagès, Paris. Given a real number $r > 0$ and a natural number n the optimal quantization of a Borel probability P on $\mathbb{R}^d$ equipped with an arbitrary norm $||.||$ deals with the n-th quantization error

$$e_{n,r} = \inf\{\left(\int \min_{a\in\alpha} ||x-a||dP(x)\right)^{\frac{1}{r}} : \alpha \subset \mathbb{R}^d \text{ and } \mathrm{card}(\alpha) \leq n\}$$

and n-optimal codebooks $\alpha \subset \mathbb{R}^d$, i.e., subsets α_n of $\mathbb{R}^d$ with at most n elements and

$$e_{n,r} = \left(\int \min_{a\in\alpha} ||x-a||dP(x)\right)^{\frac{1}{r}}$$

Classical results of quantization theory show that, under a suitable moment condition on P, n-optimal codebooks always exist and that the quantization errors satisfy

$$\lim_{n\to\infty} n^{1/d} e_{n,r} \in (0,\infty)$$

provided the absolutely continuous part of P does not vanish.

In the present talk we will show for a large class of absolutely continuous probabilities P that, for every sequence $(\alpha_n)_{n\in\mathbb{N}}$ with α_n being an n-optimal codebook and every compact subset K of the interior of the support of P, there are constants $c_1, c_2, c_3, c_4 \in (0, \infty)$ with

$$c_1 \frac{1}{n} \leq P(W(a|\alpha_n)) \leq c_2 \frac{1}{n}$$

and

$$c_3 n^{-(1+\frac{r}{d})} \leq \int_{W(a|\alpha_n)} ||x-a||^r dP(x) \leq c_4 n^{-(1+\frac{r}{d})}$$

for all $a \in \alpha_n$ with $W(a|\alpha_n) \cap K \neq \emptyset$. Here

$$W(a|\alpha_n) = \{x \in \mathbb{R}^d : ||x-a|| = \min_{b \in \alpha_n} ||x-b||\}$$

is the Voronoi cell of a with respect to α_n.

This is a partial answer to a long standing open problem in quantization theory.

Aaron Hill, *Topological isomorphism for non-degenerate rank-1 systems.* University of North Texas. aaron.hill@unt.edu.

Foreman, Rudolph, and Weiss recently proved that the isomorphism relation on rank-1 measure-preserving transformations is a Borel subset of $\mathrm{Aut}(X, \mu) \times \mathrm{Aut}(X, \mu)$. It remains an open problem to give an explicit description of when two rank-1 measure-preserving transformations are measure-theoretically isomorphic. This could be called the measure-theoretic isomorphism problem for rank-1 systems.

In this talk we will discuss the topological isomorphism problem for non-degenerate rank-1 systems. We will define the space of non-degenerate rank-1 systems and give a complete metric on that space. Each non-degenerate rank-1 system is a measure-preserving transformation and a homeomorphism of Cantor space. Our main result is an explicit description of when two non-degenerate rank-1 systems are topologically isomorphic. As consequences of this description we have the following.

- Each non-degenerate rank-1 system has trivial centralizer in the group of homeomophisms of Cantor space.
- The topological isomorphism relation on the space of non-degenerate rank-1 systems is bi-reducible to E_0.
- We have an explicit description of when a non-degenerate rank-1 system is isomorphic to its inverse.

This is joint work with Su Gao.

Ka-Sing Lau, *Iterated function systems and tree structure.* Chinese University of Hong Kong. kslau@math.cuhk.edu.hk.

It is well known that a contractive iterated function system (IFS) give rise to a symbolic space. The symbolic space has a tree structure which is most useful in representing the dynamics of the iteration and the resulting attractor K(a self-similar set in our consideration). However the tree does not capture all the geometric and analytic properties of K. In this talk we will explore the concept of *augmented tree*, which incorporate more information of K onto to the tree of symbolic space. We show that under some conditions, the augmented tree is a *hyperbolic graph* in the sense of Gromov, it has a *hyperbolic boundary* in the limit, and this hyperbolic boundary is homeomorphic to K. We apply this set up to consider various problems of Lipschitz equivalence and harmonic structure of the self-similar sets.

Jack Lutz, *The Dimensions of Individual Points in Euclidean Space.* Iowa State University. `lutz@cs.iastate.edu`.

The (constructive Hausdorff) dimension of a point x in Euclidean space is the algorithmic information density of x. Roughly speaking, this is the least real number $\dim(x)$ such that $r \cdot \dim(x)$ bits suffice to specify x on a general-purpose computer with arbitrarily high precisions 2^{-r}. This talk will survey the development of this notion, its geometric meaning, and its role in the analysis of several classes of fractals. The results surveyed are the work of many investigators.

Arnold Miller, *Universal functions.* University of Wisconsin, Madison. `miller@math.wisc.edu`.

A function of two variables $F(x, y)$ is universal iff for every other function $G(x, y)$ there exists functions $h(x)$ and $k(y)$ with

$$G(x, y) = F(h(x), k(y)).$$

Sierpinski showed that assuming the continuum hypothesis there exists a Borel function $F(x, y)$ which is universal. Assuming Martin's Axiom there is a universal function of Baire class 2. A universal function cannot be of Baire class 1. Here we show that it is consistent that for each α with $2 < \alpha < \omega_1$ there is a universal function of class α but none of class $\beta < \alpha$. We show that it is consistent with ZFC that there is no universal function (Borel or not) on the reals, and we show that it is consistent that there is a universal function but no Borel universal function. We also prove some results concerning higher arity universal functions. For example, the existence of an F such that for every G there are h_1, h_2, h_3 such that for all x, y, z

$$G(x, y, z) = F(h_1(x), h_2(y), h_3(z))$$

is equivalent to the existence of a 2-ary universal F, however the existence of an F such that for every G there are h_1, h_2, h_3 such that for all x, y, z

$$G(x, y, z) = F(h_1(x, y), h_2(x, z), h_3(y, z))$$

follows from a 2-ary universal F but is strictly weaker.

This is joint work with Paul B. Larson, Juris Steprāns, and William A.R. Weiss.

Lars Olsen, *Multifractal Analysis. A brief survey.* University of St. Andrews. `lo@st-and.ac.uk`.

Fractals are highly irregular sets and fractal geometry is the study of the geometric properties of fractal sets. Examples of fractal sets are the Cantor set, graphs of continuous but nowhere differentiable functions and self-similar sets.

Similarly, highly irregular (Borel) measures are known as multifractals, and multifractal geometry refers to the study of the geometry of multifractals. Multifractal measures are abundant and include, for example, self-similar measures and Gibbs states in dynamics.

Multifractal geometry was introduced by physicists in the mid 1980's and a large number of claims were made on the basis of heuristics and physical intuition. Since the 1990'es there has been an enormous interest in the mathematical literature to determine to what extend rigorous arguments can be provided for this theory. Dan Mauldin's contributions to this study have been seminal.

This talk will present a brief and selected survey of the development of multifractal analysis.

Brandon Seward, *Group colorings and Bernoulli subflows.* University of Michigan. b.m.seward@gmail.com.

Drawing on motivation from Borel equivalence relations, topological dynamics, and symbolic dynamics, we discuss free subflows of Bernoulli flows over arbitrary countable groups. I will discuss results on the existence of free subflows, on the density and descriptive complexity of their union, and on the complexity of the equivalence relation of topological conjugacy between (free) subflows. A powerful method of construction which we call the Fundamental Method will also be discussed.

This is joint work with Su Gao and Steve Jackson.

Todor Tsankov, *The minimal flows of S_∞.* Université Paris Diderot. todor@math.jussieu.fr.

If G is a topological group, a G-flow is a compact space equipped with a continuous action of G. Of special interest are the minimal flows (those that do not admit proper subflows) because of their rich structure and the fact that any flow must contain a minimal subflow. In the case where G is locally compact, non-compact, there is a great variety of minimal flows and a classification seems to be infeasible. On the other hand, for many naturally occurring non-locally compact groups, there is only one minimal flow – a single point – and the situation trivializes. In this talk, I will concentrate on an intermediate example, that of the group of all permutations of the integers, S_∞. It turns out that S_∞ has only countably many minimal flows that can be described quite explicitly (they are all given by the logic action of S_∞ on the space of models of certain universal theories). This classification relies in an essential way on previous work of Glasner and Weiss, who had identified the universal minimal flow of S_∞, of which all other minimal flows are quotients.

Andy Yingst, *Homeomorphic and good Bernoulli trial measures.* University of South Carolina, Lancaster. yingst@mailbox.sc.edu.

Consider an experiment having finitely many possible outcomes, and repeat this experiment independently. The result is a Bernoulli trial measure, a probability measure on Cantor space. Oxtoby asked when two such measures are homeomorphic. That is, when is there a homeomorphism of Cantor space which maps one of these measures to another. Related to this question is a definition formulated by Akin: A measure on a Cantor space is good when for each clopen set, there exists a clopen set of the same measure in any open set large enough to contain one. We also examine the question of which Bernoulli trial measures are good.

Topics touched upon will include: the approximation of a real function by an integer polynomial, the reciprocal of an algebraic integer, and a Bernoulli trial measure which admits a homeomorphism equivalent to an irrational rotation.

Stan Williams, *Guass-Green Theorems for open sets with fractal boundaries generated by graph-directed constructions.* Utah State University.
stanley.williams@usu.edu.

Graph Directed Constructions are used to construct fractal sets which serve as boundaries for bounded open subsets in two and three dimensional Euclidean space. Concrete Guass-Green Theorems for these sets are developed using the construction graph of the boundary.

Abstracts of Contributed Talks

Pieter Allaart, *The Hausdorff dimension of level sets of generalized Takagi functions.* University of North Texas. allaart@unt.edu.

Takagi's continuous but nowhere differentiable function is defined by

$$T(x) = \sum_{n=0}^{\infty} \frac{1}{2^n}\phi(2^n x),$$

where $\phi(x) = \mathrm{dist}(x, \mathbb{Z})$, the distance from x to the nearest integer. Recently there has been a great deal of interest in the level sets of T, which have been shown to possess many surprising properties. In particular it is now known that the largest Hausdorff (and box-counting) dimension of any of the level sets of T is $1/2$. In this talk I will discuss the dimension of level sets of the more general functions of the form

$$f(x) = \sum_{n=0}^{\infty} \frac{r_n(x)}{2^n}\phi(2^n x),$$

where each r_n is a $\{-1, 1\}$-valued function which may jump only at points $k/2^n$, so that each term of the above series is a continuous function. For such functions, the dimension of the level sets can be considerably greater than $1/2$. I will give a sharp upper bound for this dimension, but show that the original bound of $1/2$ remains valid when each r_n is constant. Finally, I will present some dimension results for the case when the sign functions r_n are chosen at random.

Karl Backs, *Disintegration of σ-finite measures.* University of North Texas.
karl.backs@unt.edu.

A disintegration of measure is a common tool used in ergodic theory, probability, and descriptive set theory. Traditionally disintegrations of measures are considered in settings where the measures of interest are finite. However in infinite ergodic theory such a luxury may not be available. In situations where the measures are σ-finite it is desirable that a disintegration of measure satisfy certain uniformity conditions. Dorothy Maharam was interested in such uniformly σ-finite disintegrations and asked whether this condition is always attained.

In this talk I will discuss recent work by R. Daniel Mauldin, Steve Jackson, and myself in which we present an answer to Maharam's question assuming Gödel's axiom of constructibility, $\mathbf{V} = \mathbf{L}$, to construct a non-uniformly σ-finite disintegration of measure.

James Kuodo Huang, *Hilbert logic versus Boolean logic.* Association of International Uncertainty Computing U.S.A. jkuodo@yahoo.com.

Through studying Hilbert second problem, Sixth problem, logic, set theory, number theory and computer technology that leads me to think of the old question what are the mathematical foundation for the computing science, modern natural science, and social science. The most fundamental mathematical foundation for sciences probably is logic. The author would like to propose integrated logic as modern logic foundation. Integrated logic is also classified into integrated Boolean logic and Integrated Hilbert logic. The integrated Boolean logic is based on Boolean logic and classical Cantor set theory. The integrated Hilbert logic is influenced mostly by Hilbert second problem, sixth problem and modern descriptive set theory. The main difference between Boolean integrated logic and Hilbert integrated logic is Hilbert integrated logic allowing transfinite induction to be used. This paper is dedicated to Professor Mauldins retirement conference in Denton Texas in June 2012.

Kiko Kawamura, *The Takagi function revisited.* University of North Texas. kiko@unt.edu.

The Takagi function revisited. More than a century has passed since Takagi published his simple example of a continuous but nowhere differentiable function, yet Takagi's function $T(x)$. Unlike for the more famous Weierstrass function, it is easy to show that T has at no point a finite derivative; however, it does possess an infinite derivative at many points. Then, at which set of points $T(x)$ has an infinite derivative? Surprisingly, it would take 77 years for this natural question to be answered correctly. I will explain the answer which is not all that easy to guess.

Mario Roy, *Conformal graph directed Markov systems: recent advances.* York University. mroy@glendon.yorku.ca.

The last 30 years have been a period of intensive study of finite conformal iterated function systems. Dan Mauldin, with his collaborators (Susan Williams and Mariusz Urbański, among others), have contributed new results to that theory, have generalized many results to infinite systems, and have extended the theory to conformal graph directed Markov systems. During my talk, I will give an overview of some of the recent advances in that theory and look at some open questions.

Mrinal Kanti Roychowdhury, *Hausdorff and upper box dimension estimate of hyperbolic recurrent sets.* University of Texas-Pan American. roychowdhurymk@utpa.edu.

Under the open set condition we have determined the lower and upper bounds of the Hausdorff dimensions and the upper box-counting dimensions of the limit sets of hyperbolic recurrent iterated function systems in terms of the unique zeros h and H of the two pressure functions. In addition, we have estimated the bounds of h, H-dimensional Hausdorff and packing measures. The result in this paper generalizes existing results about fractal dimensions of many other iterated function systems. In my talk I will present it.

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