

SOLUTION FOR DECEMBER 2016 PROBLEM

Find all solutions of:

$$\sqrt[3]{x+9} - \sqrt[3]{x-9} = 3.$$

SOLUTION:

$$x = \pm 4\sqrt{5}.$$

Correct solutions were turned in by: William Liu, Xiangyu Kong, and Ethan Seal.

Cubing both sides gives of the equation gives:

$$x + 9 - 3\sqrt[3]{(x+9)^2}\sqrt[3]{x-9} + 3\sqrt[3]{x+9}\sqrt[3]{(x-9)^2} - (x-9) = 27.$$

Thus:

$$-\sqrt[3]{(x+9)^2}\sqrt[3]{x-9} + \sqrt[3]{x+9}\sqrt[3]{(x-9)^2} = 3.$$

Rewriting this we get:

$$-\sqrt[3]{x+9}\sqrt[3]{x-9} (\sqrt[3]{x+9} - \sqrt[3]{x-9}) = 3.$$

Now notice that the term in the parenthesis is 3 (look at the original statement of the problem).

Thus:

$$\sqrt[3]{x+9}\sqrt[3]{x-9} = -1.$$

Cubing both sides again gives:

$$x^2 - 81 = -1.$$

Therefore:

$$x^2 = 80 = 16 \cdot 5.$$

Thus:

$$x = \pm 4\sqrt{5}.$$