SOLUTION FOR MARCH 2017 PROBLEM

SOLUTION:

128462

We begin with:

$$\begin{array}{cccccc} & M & A & J & O & R \\ + & M & I & N & O & R \\ = & R & E & S & U & M & E \end{array}$$

We first see from the second column that $M+M$ is an even number between 0 and 18 so either $R=0$ or $R=1$. But $R \neq 0$ for if $R=0$ then the sixth column would give $E=0$ but $E \neq R$. Thus $R=1$. Then from the sixth column we see $E=2$. From the first and second column we see that $M+M=12$ so $M=6$. From the fifth column we see that either $O=3$ or $O=8$.

CASE 1: $O=3$

If $O=3$ then the remaining digits are $0,4,5,7,8,9$ and we have:

$$\begin{array}{cccccc} & 6 & A & J & 3 & 1 \\ + & 6 & I & N & 3 & 1 \\ = & 1 & 2 & S & U & 6 & 2 \end{array}$$

Next one possibility is $J+N=U$ and $A+I=S$. However notice that $S \neq 0$ and $U \neq 0$. Also notice that $J \neq 0$ for if so then $N=U$ but different letters correspond to different numbers. Similarly $N \neq 0$, $A \neq 0$, and $I \neq 0$.

Thus we see that $J+N$ is greater than or equal to 10 and so we get $J+N-10=U$. Since $J+N \geq 10$ then we see that the other equation must be $1+A+I=S$. So our two remaining equations are:

$$J+N=U+10$$

$$A+I=S-1.$$

Next we notice $J \neq 0$ for if $J=0$ then $N=U+10$ but $0 \leq N \leq 9$ and $U+10 \geq 10$. Similarly $N \neq 0$ and $S \neq 0$. Thus either $A=0$ or $I=0$.

SUBCASE i: $A=0$

If $A=0$ then we have:

$$J+N=U+10$$

$$1+I=S$$

and the remaining digits are: $4,5,7,8,9$. This forces $U=4,5$, or 7. (If $U=9$ then $J+N=19$ but among the numbers $0,4,5,7,8$ no two of these sum to 19 so $U \neq 9$. Similarly $U \neq 8$).

If $U=4$ then J and N need to be 5 and 9 (or vice versa). This leaves 7 and 8 for I and S which forces $S=8$ and $I=7$. Thus we end up with RESUME= 128462.

If $U = 5$ then J and N need to be 7 and 8 (or vice versa). This leaves 4 and 9 and $1 + I = S$ which is impossible.

If $U = 7$ then J and N need to be 8 and 9 (or vice versa). Thus we see $I = 4$ and $S = 5$ which gives RESUME=125762.

SUBCASE ii: $I = 0$

A similar analysis yields $U = 4, 5$, or 7 and again the largest number we obtain for RESUME is 128462.

CASE 2: $O = 8$

If $O = 8$ then the remaining digits are $0, 3, 4, 5, 7, 9$ and we have:

$$\begin{array}{cccccc} & 6 & A & J & 8 & 1 \\ + & 6 & I & N & 8 & 1 \\ = 1 & 2 & S & U & 6 & 2 \end{array}$$

This leave two cases.

SUBCASE i:

$$J + N + 1 = U$$
$$A + I = S$$

Now the only way for RESUME to be larger than 128462 is for $S = 9$ so we try to rule out this possibility. If $S = 9$ then we see that $A = 4$ and $I = 5$ (or vice versa). This leaves the digits $0, 3$ and 7 and $J + N + 1 = U$ which is impossible.

SUBCASE ii:

$$J + N + 1 = U + 10$$
$$A + I + 1 = S.$$

Similarly we try to rule out $S = 9$. If $S = 9$ then we see that $A = 3$ and $I = 5$ (or vice versa). This leaves the digits $0, 4, 7$ and $J + N = U + 9$ which is again impossible.

Thus the largest number for RESUME is 128462.