SOLUTION FOR DECEMBER 2019

Correct solutions were submitted by:

Aryan Agarwal
Rhythm Garg
Chris Howard
Arnav Iyer
Andrew Sansom
Weilun Sun
Hubert Yang

SOLUTION: Prove the following:

$$\frac{1}{3} = \frac{1+3}{5+7} = \frac{1+3+5}{7+9+11} = \frac{1+3+5+7}{9+11+13+15} = \cdots . \tag{1}$$

Proof: Notice that the numerator is the sum of the first n odd numbers and the denominator is the sum of the next n odd numbers where $n = 1, 2, 3, ...$

Also it is a well-known fact (and it can be shown by induction) that the sum of the first n odd numbers is n^2. That is:

$$\sum_{k=1}^{n}(2k-1) = n^2. \tag{2}$$

Now the denominator is just the sum of the next n odd numbers which is:

$$\sum_{k=n+1}^{2n}(2k-1) = \sum_{k=1}^{2n}(2k-1) - \sum_{k=1}^{n}(2k-1). \tag{3}$$

It follows then by (2) that:

$$\sum_{k=1}^{2n}(2k-1) - \sum_{k=1}^{n}(2k-1) = (2n)^2 - n^2 = 4n^2 - n^2 = 3n^2. \tag{4}$$

Finally combining (2) and (3)-(4) we see that:

$$\frac{\sum_{k=1}^{n}(2k-1)}{\sum_{k=n+1}^{2n}(2k-1)} = \frac{n^2}{3n^2} = \frac{1}{3}.$$

□