SOLUTION FOR FEBRUARY 2019

Solution: Prove that $r = A'E$.

Proof: Let P be the point between A and D where paper was folded and Q is the point between B and C where paper was folded. Let $z = AB, x = AP$. So $PA' = x, A'E = w, PE = \sqrt{x^2+w^2}, EB' = z - w$. So we want to show $r = w$.

Next note that the triangles $PA'E$, $B'DE$ and QCB' are all similar. Since the first two are similar then we see $ED = cw, DB' = cx, EB' = c\sqrt{x^2+w^2}$ for some $c > 0$.

Next since the length of the segment AB is the same as the length of the segment AD then we have:

$$z = x + \sqrt{x^2+w^2} + cw.$$

Also since the length of the segment AB is the same as the length of the segment $A'B'$ then we have:

$$w + c\sqrt{x^2+w^2} = z.$$

Equating these and solving for c gives:

$$c = \frac{x}{\sqrt{x^2+w^2} - w} + 1.$$

Multiplying by $\frac{\sqrt{x^2+w^2}+w}{\sqrt{x^2+w^2}+w}$ gives:

$$c = \frac{\sqrt{x^2+w^2}+w}{x} + 1$$

so:

$$cx = \sqrt{x^2+w^2} + w + x. \tag{1}$$

Now using the fact mentioned in the hint that:

$$r = \frac{2\text{Area}(B\text{'}DE)}{\text{Perimeter}(B\text{'}DE)}$$

gives:

$$r = \frac{c^2xw}{c(\sqrt{x^2+w^2}+x+w)} = \frac{cxw}{\sqrt{x^2+w^2}+x+w}. \tag{2}$$

Finally by (1)-(2) we get:

$$r = \left(\frac{cx}{\sqrt{x^2+w^2}+x+w}\right)w = 1 \cdot w = w.$$

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