

### SOLUTION FOR APRIL 2019

Correct solutions were submitted by:

Rhythm Garg  
Jingyi Dai  
Eduardo Luna

**Solution:** Let:

$$xy = z + w$$

$$zw = x + y.$$

$x, y, z, w$  positive integers

Suppose  $x \geq y$ ,  $z \geq w$  and  $x \geq z$ . Then since  $y \geq 1$  we have from first eq

$$xy \leq 2x$$

therefore  $y \leq 2$ . Since  $y > 0$  then either  $y = 1$  or  $y = 2$ . If  $y = 2$  then we end up with  $(2z - 1)(2w - 1) = 9$ . Since  $z \geq w$  we get  $2z - 1 = 9$  and  $2w - 1 = 1$  OR  $2z - 1 = 2w - 1 = 3$ . In the first instance  $z = 5$ ,  $w = 1$ , and so  $x = 3$  but  $x \geq z$  so this doesn't happen. In the second we get  $z = w = 2$  and thus  $x = 2$ .

If  $y = 1$  we get  $(z - 1)(w - 1) = 2$ . Since  $z \geq w$  we get  $z - 1 = 2$  and  $w - 1 = 1$  so  $z = 3$ ,  $w = 2$  and so  $x = 5$ .