

SOLUTION FOR OCTOBER 2020

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Show that none of the numbers 11, 111, 1111, 11111, etc. is a perfect square.

SOLUTION: Notice that $11 = 4(2) + 3$, $111 = 4(27) + 3$, $1111 = 4(277) + 3$, etc. That is 11, 111, 1111, etc. all have remainder 3 when divided by 4. Thus they are all of the form $4n + 3$ where n is an integer. However the square of an even number, $2n$, is $4n^2$ and the square of an odd number, $2n + 1$, is $4n^2 + 4n + 1 = 4(n^2 + n) + 1 = 4m + 1$ with m an integer so that the square of a number is either a multiple of 4 or has remainder 1 when divided by 4.

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