SOLUTION FOR APRIL 2022

A correct solution was submitted by:

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Let T be a right triangle with sides a, b, and hypotenuse c. Draw the incircle with radius r. Denote the vertices as L, Q, R with vertex Q at the right angle. Let S be the intersection where the incircle meets the hypotenuse. Show that:

$$LS = \frac{1}{2}(c + a - b).$$

HINT: It is known that if A is the area and P the perimeter then $A = \frac{1}{2}rP$.

SOLUTION: Using $A = \frac{1}{2}rP$ gives $\frac{1}{2}ab = A = \frac{1}{2}rP = \frac{1}{2}r(a + b + c)$. Thus $ab = r(a + b + c)$. At the right angle we have a square with side r. Let X be the center of the incircle. And denote the square with side r as $QUXV$ where U is on LQ and V is on QR .Then LUX is congruent to LSX and also RVX is congruent to RSX. This then implies $a - r + b - r = c$ so:

$$a + b - c = 2r. \tag{1}$$

Next we have $LS = a - r$ and so using (1) gives $2LS = 2a - 2r = 2a - (a + b - c) = c + a - b$ and thus:

$$LS = \frac{1}{2}(c + a - b).$$