## SOLUTION FOR FEBRUARY 2025

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**Problem:** Let $a_1a_2a_3$ be any 3 digit number where each of $a_i$ is an integer with $0 \leq a \leq 9$. Next perform the following. Write the digits in decreasing order then write the digits in increasing order and then subtract these two numbers. Define $x_1$ to be this result. For example, if my original number number is 001 then $x_1 = 100 - 001 = 099$. Now repeat this process with $x_1$ and get another number $x_2$. So if $x_1 = 099$ then $x_2 = 990 - 099 = 891$. Continue this process and show that $x_6 = 0$ or 495. In addition, show that the number of iterations cannot be decreased. That is, find an $a_1a_2a_3$ for which $x_5 \neq 0$ and $x_5 \neq 495$.

**Proof:** Take the given number $a_1a_2a_3$ and then rewrite this so that the numbers are decreasing. Let's call this $abc$ where $a \geq b \geq c$.

Now the number $abc$ is really $100a + 10b + c$ and $cab$ is really $100c + 10b + a$. Then

$$x_1 = (100a + 10b + c) - (100c + 10b + a) = 99(a - c).$$

If $a = c$ then $x_1 = 0$ and it follows that $x_n = 0$ for all $n \geq 1$. Otherwise, $x_1$ is a (positive) multiple of 99 with 3 (or 2) digits. So

$$x_1 = 99, 198, 297, 396, 495, 594, 693, 792, \text{ or } 891.$$

Now if we write these so that the digits are in decreasing order then we have either $990, 981, 972, 963$, or 954. Thus $x_2 = 891, 792, 693, 594$, or 495. Writing these in decreasing order gives either $981, 972, 963$, or 954. Then $x_3 = 792, 693, 594$, or 495. Writing in decreasing order again gives either $972, 963$, or 954. Then $x_4 = 693, 594$, or 495. Then $x_5 = 594$ or 495. Finally $x_6 = 495$.

To find a 3 digit number that satisfies $x_5 \neq 495$ try beginning with 001. Then $x_1 = 099$, $x_2 = 891, x_3 = 792, x_4 = 693, x_5 = 594$, and $x_6 = 495$.

A similar result is true for 4 digit numbers. Can you figure out what happens in that situation?