SOLUTION FOR OCTOBER 2015

Let T be an equilateral triangle. Let P be an arbitrary point in T. Let d_1, d_2, and d_3 be the distances of P to each of the three sides of T. Show that $d_1 + d_2 + d_3$ is independent of P! That is, show that $d_1 + d_2 + d_3$ is the same regardless of which point P is chosen.

Solution:

$$d_1 + d_2 + d_3 = \frac{\sqrt{3}}{2}s$$

where s is the length of a side of the triangle.

There were several correct solutions turned in this month. Nice going everyone! A few of them gave an argument similar to the following.

Let P be an arbitrary point in the equilateral triangle XYZ where s is the length of a side of the triangle. Then it is known that the area of triangle XYZ is $\frac{\sqrt{3}}{4}s^2$. Next we drop a perpendicular of length d_1 from P to say side YZ. Calculating the area of PYZ we see we get $\frac{1}{2}sd_1$. Similarly we drop a perpendicular of length d_2 from P to XY. Calculating the area of PXY we obtain $\frac{1}{2}sd_2$. Finally we do the same with d_3 and triangle PXZ. Its area is $\frac{1}{2}sd_3$. Adding the areas of the three small triangles we see that this is equal to the area of XYZ which is $\frac{\sqrt{3}}{4}s^2$. Thus we obtain: $\frac{\sqrt{3}}{4}s^2 = \frac{1}{2}s(d_1 + d_2 + d_3)$. After rewriting we obtain:

$$d_1 + d_2 + d_3 = \frac{\sqrt{3}}{2}s.$$