

### SOLUTION FOR MAY 2024

Determine which of the following numbers is a perfect square 1, 14, 144, 1444, 14444,  $\dots$ .

**Solution** The only numbers which are perfect squares are  $1 = 1^2$ ,  $144 = 12^2$ ,  $1444 = 38^2$ .

**Proof:** We know that 14 is not a perfect square and that 144 and 1444 are perfect squares so now let us consider 14444. A similar proof works for the other numbers.

Now suppose there is an  $n$  such that  $n^2 = 14444$ . Then  $n^2$  is even and so therefore  $n = 2k$  for some  $k$  and therefore  $4k^2 = 14444$  and thus  $k^2 = 3611$ .

Now  $k^2$  is odd so therefore  $k$  must be odd and so  $k = 2m + 1$ . Thus  $4m^2 + 4m + 1 = k^2 = 3611$  and thus  $4(m^2 + m) = 3610$  therefore  $2(m^2 + m) = 1805$  but the left-hand side is even and the right-hand side is odd. Thus we obtain a contradiction.