# A characterization of compactness for singular integral operators

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Broadly speaking, Calderón-Zygmund operators can be defined as integrals of the form

$$Tf(x) = \int_{\mathbb{R}^d} f(t)K(t,x)dt,$$

where $x \notin \sup f$, and $K$ is a kernel that may be singular along the diagonal, namely, $|K(t,x)| \lesssim |t-x|^{-\alpha}$, with $0 < \alpha \leq d$.

Relevant examples include the Cauchy transform in Complex Analysis, the Hilbert and Riesz transforms in Real Harmonic Analysis, and Double Layer potential operators in elliptic PDEs. The latter case provide the solution to the Dirichlet problem associated with the Laplace equation.

The classical $T1$ Theorem of David and Journé is a characterization of boundedness for C-Z operators. It can be used to prove existence of solution to the Laplace equation by showing compactness of the corresponding Double Layer potential operator, and applying Fredholm Theory. For this, the traditional approach is first to use the $T1$ Theory to establish boundedness, and then develop some ad hoc work to deduce compactness.

To solve the Laplacian more effectively, we introduce a new $T1$ Theory to characterize those C-Z operators that extend compactly on $L^p(\mathbb{R}^d)$ for $1 < p < \infty$. In line with David and Journé's seminal paper, the conditions for compactness are expressed in terms of the decay of the kernel derivative and the action of the operator over special families of functions.

We apply our results to study compactness of Double Layer Potential operators associated with non-smooth domains, and Cauchy transforms over Radon measures of power growth.